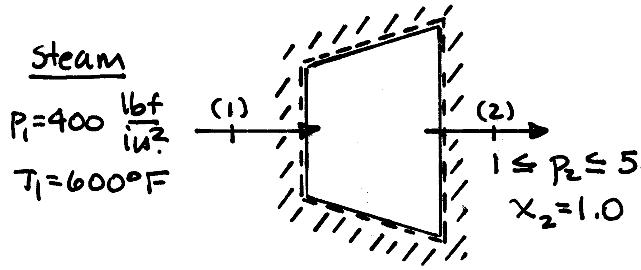
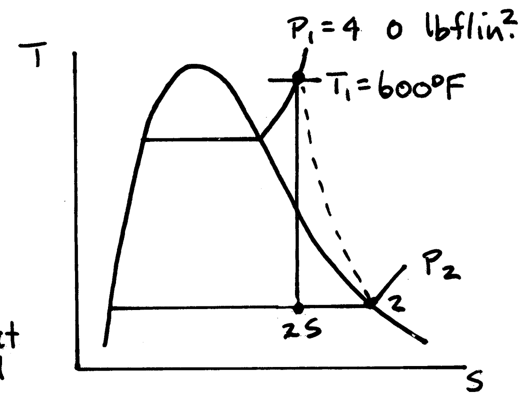
PROBLEM 7.100

KNOWN: Steam enters a well-insulated turbine operating at steady state at a specified state and exits at pressure p.

FIND: (a) For p = 50 lbf/in², determine the exergy destruction rate per unit mass of steam flowing and the isentropic and exergetic turbine efficiencies. (b) Plot each of these quantities versus p ranging from 1 to 50 lbf/in²

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) The control volume shown operates at steady state. (2) For the control volume, $\dot{Q}_{cv} = 0$ and kinetic and potential energy effects are ignored. (3) For the environment, $T_0 = 60°F = 520°R$, $P_0 = 1$ atm.

ANALYSIS: With assumptions 1 and 2, the mass and energy rate balances reduce to give

$$\frac{\dot{W}_{cv}}{\dot{m}} = h_1 - h_2 \tag{1}$$

Further, mass and entropy rate balances reduce to give $\dot{\sigma}_{cv}/\dot{m} = s_2 - s_1$. The exergy destruction rate is $\dot{E}_d/\dot{m} = T_0(\dot{\sigma}_{cv}/\dot{m})$, or

$$\dot{E}_d/\dot{m} = T_0(s_2 - s_1) \tag{2}$$

Furthermore, the isentropic and exergetic turbine efficiencies are, respectively

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \tag{3}$$

$$\epsilon = \frac{\dot{W}_{cv}/\dot{m}}{e_{f1} - e_{f2}} = \frac{h_1 - h_2}{(h_1 - h_2) - T_0(s_1 - s_2)} \tag{4}$$

(a) From Table A-4E, $h_1 = 1306.6$ Btu/lb, $s_1 = 1.5892$ Btu/lb·°R. Further, from Table A-3E at 50 lbf/in², $h_2 = h_g = 1174.4$ Btu/lb, $s_2 = s_g = 1.6589$ Btu/lb·°R. Accordingly

$$\dot{E}_d/\dot{m} = (520°R)(1.6589 - 1.5892)\frac{Btu}{lb \cdot °R} = 36.24\ Btu/lb \longleftarrow \dot{E}_d/\dot{m}$$

State 2s is fixed by p_2 and $s_{2s} = s_1$: $x_{2s} = (s_{2s} - s_f)/s_{fg} = (1.5892 - 0.4113)/1.2476 = 0.9441$. Therefore $h_{2s} = h_f + x_{2s} h_{fg} = 250.24 + (.9441)(924.2) = 1122.8$ Btu/lb.

Thus

$$\eta_t = \frac{1306.6 - 1174.4}{1306.6 - 1122.8} = 0.719\ (71.9\%) \longleftarrow \eta_t$$

Now, from Eq. (4)

$$\epsilon = \frac{(1306.6 - 1174.4)}{(1306.6 - 1174.4) - (520)(1.5892 - 1.6589)} = 0.785\ (78.5\%) \longleftarrow \epsilon$$

PROBLEM 7.100 (Cont'd.)

(b) The data for the required plots are obtained using IT, as follows:

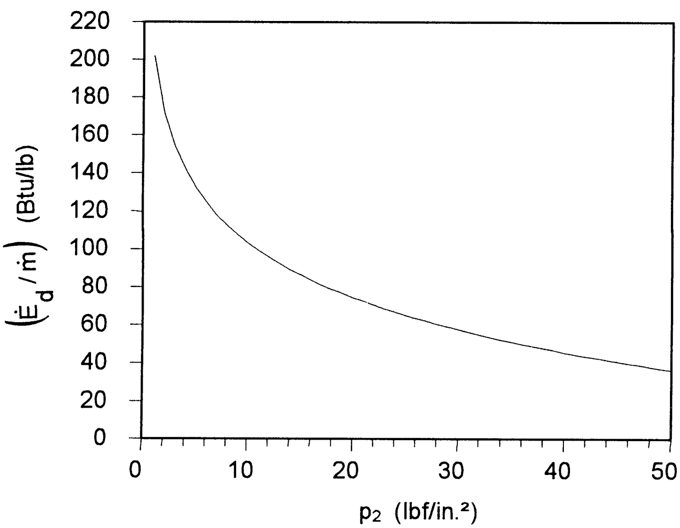
IT Code

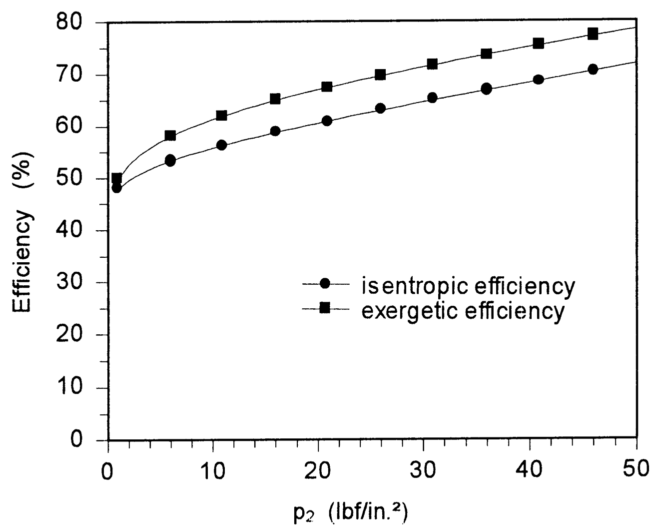
```
p1 = 400  // lbf/in.²
T1 = 600  // °F
x2 = 1.0
p2 = 50  // lbf/in.²
To = 60 + 460  // °R

h1 = h_PT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2 = hsat_Px("Water/Steam", p2, x2)
s2 = ssat_Px("Water/Steam", p2, x2)
s2s = s_Ph("Water/Steam", p2, h2s)
s2s = s1

Ed = To * (s2 - s1)
eta = (h1 - h2) / (h1 - h2s)
eff = (h1 - h2) / ((h1 - h2) - To * (s1  -s2))
```

IT Results for p_2 = 50 lbf/in.²

h_1 = 1306 Btu/lb
h_2 = 1174 Btu/lb
h_{2s} = 1123 Btu/lb
s_1 = 1.589 Btu/lb·°R
s_2 = 1.659 Btu/lb·°R
s_{2s} = 1.589 Btu/lb·°R
$\dot{E}_d / \dot{m}$ = 36.26
η_t = 0.719 (71.9%)
ε = 0.7847 (78.47%)

PLOTS:





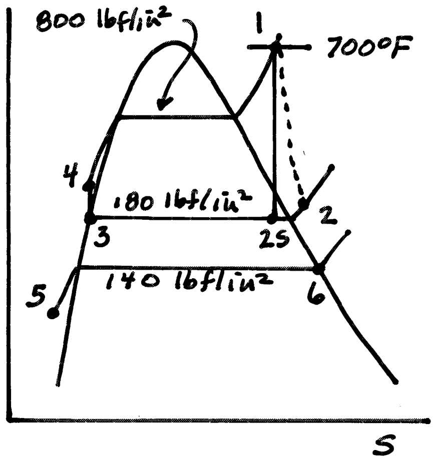
As expected, increased efficiency corresponds with decreased exergy destruction.

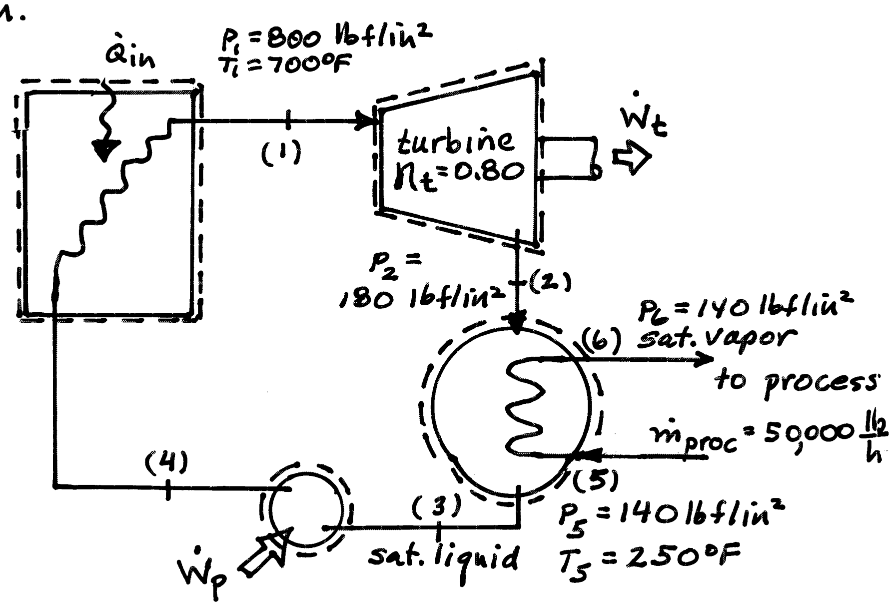
PROBLEM 8.76

KNOWN: Water is the working fluid in a Rankine cycle used for cogeneration. The cycle produces power, and energy rejected from the condensing steam is transferred to a separate process stream. Data are known at various locations.

FIND: Determine the mass flow rate for the Rankine cycle working fluid, and devise and evaluate an exergetic efficiency for the overall cogeneration system.

SCHEMATIC & GIVEN DATA:





ASSUMPTIONS: (1) Each component is analyzed as a control volume at steady state. (2) The turbine, pump, and condenser operate adiabatically. (3) For the turbine, $\eta_t = 0.80$, and the pump is assumed to operate isentropically. (4) Kinetic and potential energy effects are negligible. (5) Saturated liquid exits the condenser at the condenser pressure. (6) $T_0 = 530°R$, $p_0 = 14.7$ lbf/in².

ANALYSIS: First, fix each of the principal states.

State 1: $p_1 = 800$ lbf/in², $T_1 = 700°F \Rightarrow h_1 = 1338.0$ Btu/lb, $s_1 = 1.5471$ Btu/lb·°R

State 2: First, $p_2 = 180$ lbf/in², $s_{2s} = s_1 \Rightarrow h_{2s} = 1191.1$ Btu/lb. With the turbine efficiency

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow h_2 = h_1 - \eta_t (h_1 - h_{2s}) = 1220.5 \text{ Btu/lb}$$

and, from Table A-4E; $s_2 \approx 1.5818$ Btu/lb·°R

State 3: $p_3 = 180$ lbf/in², sat. liquid $\Rightarrow h_3 = 346.3$ Btu/lb, $s_3 = 0.5329 \frac{\text{Btu}}{\text{lb}\cdot°\text{R}}$

State 4:
$$h_4 \approx h_3 + v_4 (p_4 - p_3)$$
$$= 346.3 \frac{\text{Btu}}{\text{lb}} + \left(0.01827 \frac{\text{ft}^3}{\text{lb}}\right)(800-180)\frac{\text{lbf}}{\text{in}^2}\left|\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right|\left|\frac{1 \text{ Btu}}{778 \text{ ft}\cdot\text{lbf}}\right|$$
$$= 346.3 + 2.10 = 348.4 \text{ Btu/lb}, \quad s_4 = s_3 = 0.5329 \text{ Btu/lb}\cdot°\text{R}$$

State 5: $p_5 = 140$ lbf/in², $T_5 = 250°F \Rightarrow$ compressed liquid. Thus

$$h_5 \approx h_f(250) + v_f(250)\left[p - p_{sat@250°F}\right]$$
$$= 218.6 + (0.01700)(140 - 29.82)\left|\frac{144}{778}\right| = 218.9 \text{ Btu/lb}$$

and $s_5 \approx s_f(250) = 0.3677$ Btu/lb·°R

State 6: $p_6 = 140\ lbf/in^2$, sat. vapor $\Rightarrow h_6 = 1193.8\ Btu/lb$, $s_6 = 1.5761\ Btu/lb\cdot°R$

To find the cycle mass flow rate, use an energy balance for the condenser

$$0 = \dot{m}_{cycle}(h_2 - h_3) + \dot{m}_{proc}(h_5 - h_6)$$

or

$$\dot{m}_{cycle} = \dot{m}_{proc}\left(\frac{h_5 - h_6}{h_3 - h_2}\right)$$

$$= (50{,}000\ lb/h)\left(\frac{218.9 - 1193.8}{346.3 - 1220.5}\right) = 5.577 \times 10^4\ lb/h \longleftarrow \underline{\dot{m}_{cycle}}$$

For the cogeneration system, the cycle net work and the exergy transferred to the process stream are the <u>outputs</u>, and the exergy increase of the working fluid passing through the steam generator is the <u>input</u>. The difference between the input and output represents exergy <u>destroyed</u> due to irreversibilities. Thus, a reasonable exergetic efficiency is

$$\epsilon = \frac{\text{net rate of exergy output}}{\text{rate of exergy input}}$$

$$= \frac{\dot{W}_{cycle} + \dot{m}_{proc}(e_{f6} - e_{f5})}{\dot{m}_{cycle}(e_{f1} - e_{f4})} \longleftarrow \underline{\epsilon\ (\text{definition})}$$

Evaluating the various quantities in this expression

$$\dot{W}_{cycle} = \dot{W}_t - \dot{W}_p = \dot{m}_{cycle}[(h_1 - h_2) - (h_4 - h_3)]$$

$$= (5.577 \times 10^4)[(1338.0 - 1220.5) - (2.10)] = 6.436 \times 10^6\ Btu/h$$

$$e_{f6} - e_{f5} = (h_6 - h_5) - T_0(s_6 - s_5)$$

$$= (1193.8 - 218.9) - (530)(1.5761 - 0.3677) = 334.4\ Btu/lb$$

$$e_{f1} - e_{f4} = (h_1 - h_4) - T_0(s_1 - s_4)$$

$$= (1338.0 - 348.4) - (530)(1.5471 - 0.5329) = 452.1\ Btu/lb$$

Thus

$$\epsilon = \frac{6.436 \times 10^6\ Btu/h + (5.577 \times 10^4\ lb/h)(334.4\ Btu/lb)}{(5.577 \times 10^4\ lb/h)(452.1\ Btu/lb)}\ \times$$

$$= \frac{6.436 \times 10^6 + 1.865 \times 10^6}{2.521 \times 10^7} = 0.329\ (32.9\%)\ \times \longleftarrow \underline{\epsilon\ (\text{value})}$$

See attached I.T.

Solution of M&S 8-76 with IT Software

```
//Turbine
0 =  - wt + (h1 - h2)
T1=700
p1=800
p2=180
etat=0.8 // Isentropic efficiency
h1 = h_pT("Water/Steam", p1, T1)
s1 = s_PT("Water/Steam", p1, T1)
h2_s = h_Ps("Water/Steam",p2, s1)
0 = - w_s + (h1 - h2_s)  //Energy-Entropy balances
etat * w_s = wt
T2 = T_Ph("Water/Steam", p2, h2)
s2 = s_PT("Water/Steam", p2, T2)
//**********************************************************
//Condenser, e refers to external stream
medot=50000
0=medot*(he1-he2)+mdot*(h2-h3)
Te1=250
pe1=140
pe2=140 // sat. vap
xe1=0 // compressed liquid
he1 = h_PT("Water/Steam", pe1, Te1)
xe2=1
he2 = hsat_Px("Water/Steam", pe2, xe2)
se1= s_PT("Water/Steam", pe1, Te1) //for exergy calc below
se2 = ssat_Px("Water/Steam", pe2, xe2) //.. .. .. ..
x3=0  //assume sat. liquid at outlet of condenser and p3=p2
h3 = hsat_Px("Water/Steam", p2, x3)
s3 = ssat_Px("Water/Steam", p2, x3)
 //***********************************************************************
//Feedwater pump
wp=vsat_Px("Water/Steam", p2, x3)*(p1-p2) //  incompressible liquid
deltah=v*deltap
h4=h3 +wp
s4=s3
wnet=wt-wp
//*********************************************************************
//Exergic efficiency
//T0=530
eff=( wnet*mdot + medot*(  he2-he1 - 530*(se2-se1) ) )/ mdot*((h1-h4)
-530*(s1-s4))
```

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```
T2 = T_Ph("Water/Steam", p2, h2)
s2 = s_PT("Water/Steam", p2, T2)
//*************************************************************
//Condenser, e refers to external stream
medot=50000
0=medot*(he1-he2)+mdot*(h2-h3)
Te1=250
pe1=140
pe2=140 // sat. vap
xe1=0 // compressed liquid
he1 = h_PT("Water/Steam", pe1, Te1)
xe2=1
he2 = hsat_Px("Water/Steam", pe2, xe2)
se1= s_PT("Water/Steam", pe1, Te1) //for exergy calc k
se2 = ssat_Px("Water/Steam", pe2, xe2) //.. .. .. ..
x3=0  //assume sat. liquid at outlet of condenser and p3
h3 = hsat_Px("Water/Steam", p2, x3)
s3 = ssat_Px("Water/Steam", p2, x3)
//************************************************************************
//Feedwater pump
wp=vsat_Px("Water/Steam", p2, x3)*(p1-p2) //  incompr
h4=h3 +wp
s4=s3
wnet=wt-wp
//************************************************************************
//Exergic efficiency
//T0=530
eff=( wnet*mdot + medot*(  he2-he1 - 530*(se2-se1) ) )
```

	1
T2	410
eff	0.9176
h1	1338
h2	1220
h2_s	1191
h3	346.2
h4	357.5
he1	218.6
he2	1194
mdot	5.578E4
s1	1.547
s2	1.582
s3	0.5328
s4	0.5328
se1	0.3677
se2	1.576
w_s	147
wnet	106.3
wp	11.33
wt	117.6
T1	700
Te1	250
etat	0.8
medot	5E4
p1	800
p2	180
pe1	140
pe2	140
x3	0