Earnings Components, Accounting Bias and Equity Valuation

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Abstract

In this paper we address three issues in accounting-based equity valuation: (i) How are valuation parameters related to earnings persistence and accounting conservatism when earnings components aggregate, or "add up", in valuation? (ii) What does aggregation of earnings components in valuation imply for abnormal earnings dynamics? and (iii) When is an earnings component "irrelevant" and "core" earnings the relevant construct for valuation? Assuming linear valuation, no-arbitrage, dividend irrelevance and clean surplus accounting, we show that when earnings components aggregate, valuation expressions and abnormal earnings dynamics are generalizations of the Ohlson (1995) model, incorporating simple adjustments for accounting conservatism. When "core" earnings is the relevant earnings construct, valuation expressions closely resemble the aggregation case, but core (abnormal) earnings replaces clean surplus (abnormal) earnings. We demonstrate that an earnings component can be irrelevant in valuation even when it is predictable.

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Financial analysts are known to take account of earnings "quality" differences when defining earnings multiples and capitalization factors in equity valuation. This raises the question of how valuation multiples are linked to accounting properties such as conservatism and earnings persistence. Firms also disclose numerous potentially informative income statement line items in financial statements, raising questions concerning the properties of accounting numbers that will lead to earnings components "adding up" (or aggregating) to a summary earnings number that is useful in valuation. Finally, despite frequent focus on "bottom line" earnings, analysts sometimes choose to ignore earnings components (e.g. special items) when forecasting future earnings and when estimating equity values. This raises questions about the properties of earnings components that can safely be disregarded in valuation. This paper addresses each of these questions using a simple linear valuation model with two earnings components.

We analyze a model where accounting can be conservative, such that book value is expected asymptotically to be less than market value. The analysis is related to the Ohlson (1995) unbiased accounting model.[1] We show that simple modifications to allow for accounting conservatism can be applied to the valuation expressions in Ohlson (1995). Whereas in Ohlson (1995) the market value of the firm equals book equity value plus a multiple of abnormal earnings, the analogous valuation expression in our model also includes conservatism "premium", equal to a multiple of lagged book value. Interpreted another way, the conservatism premium is equivalent to an effective reduction to the capital charge employed in computing abnormal earnings. We also show that similar to Ohlson (1995), the value of the firm can be expressed as a weighted average of a "flow" measure - equal to capitalized earnings adjusted for dividends, and a "stock" measure equal to book value. However, in our model the valuation weights and the capitalization factor applied to earnings incorporate conservatism adjustments.

Our analysis makes explicit links between the informational roles of accounting items in valuation and their informational roles in forecasting abnormal earnings. We show that when earnings components aggregate in valuation, they also aggregate in forecasting, whether or not accounting is conservative. Further, in contrast to the abnormal earnings dynamics assumed in Feltham and Ohlson (1995, 1996), *both* current and lagged book value (or equivalently dividends) have roles in predicting abnormal earnings when accounting is conservative. When earnings components aggregate, a single parameter in the abnormal earnings dynamics captures the degree of accounting conservatism, and this parameter is linked to the parameter controlling conservatism adjustments in valuation expressions. The displacement effect of dividends on future earnings is also directly linked to this conservatism parameter.

We also conduct analysis related to linear information models where a component of earnings is irrelevant in valuation. Stark (1997) shows that an earnings component is irrelevant in valuation if it has no predictive ability for other accounting items. Ohlson (1999) presents a model where a "transitory" earnings component is irrelevant in valuation if it is irrelevant in forecasting abnormal earnings and if it is, itself, unpredictable. In both these cases a "core" earnings construct becomes the focus for valuation. Our analysis is similar in spirit but with some differences. In contrast to Stark (1997) the irrelevant earnings component may have predictive ability for other accounting items, including abnormal earnings. In contrast to Ohlson (1999), the irrelevant earnings component is not necessarily unpredictable. We establish valuation expressions closely resembling the aggregation case, but based on core (abnormal) earnings.

The remainder of the paper is organized as follows: in Section 1 we describe the model and its general properties; Section 2 contains our analysis of earnings component aggregation; in Section 3 we consider the properties of accounting where one earnings component is irrelevant in valuation; and finally Section 4 we conclude.

1 The Basic Model

Define the set of accounting items in period t financial statements as $Z_t = \{x_{1t}, x_{2t}, b_t, d_t\}$, where x_{1t} and x_{2t} are two earnings components summing to aggregate earnings (or comprehensive income), x_t ($\equiv x_{1t} + x_{2t}$); b_t is book value at the end of period t; and d_t is dividends paid (net of new equity contributions). Our main results rely on four basic assumptions.[2] We follow Ohlson (1995, 1999) and Feltham and Ohlson (1995) in making two standard initial assumptions.

A1. The firm is valued in a risk-neutral, arbitrage-free market with $E_t[P_{t+1}+d_{t+1}] = RP_t$, where P_t is the value of the firm at the end of period t and R equals one plus the constant cost of equity capital and $E_t[.]$ is the expectations operator based on information available to time t. This assumption leads to the well-known dividend discount valuation model:

$$P_t = \sum_{s=1}^{\infty} R^{-s} E_t[d_{t+s}]. \qquad \text{(DDM)}$$

A2. In order to link financial statements inter-temporally we assume that the clean surplus accounting relation holds:

$$b_t = b_{t-1} + x_{1t} + x_{2t} - d_t. \qquad \text{(CSR)}$$

This assumption states that all changes in the book value of equity flow through one of the two earnings components. Similar to Ohlson (1995), we introduce three mathematical restrictions on CSR originating in the accounting for owners' equity: (i) $\partial b_t / \partial d_t = -1$; (ii) $\partial x_{1t} / \partial d_t = 0$; and (iii) $\partial x_{2t} / \partial d_t = 0$. These restrictions require that dividends reduce the book value of the assets held by the firm dollar for dollar but do not affect contemporaneous earnings or components of earnings.[3]

Edwards and Bell (1961), Peasnell (1982) Ohlson (1989, 1995) and others show that assumptions A1 and A2 lead directly to the well-known the residual income valuation model:

$$P_t = b_t + \sum_{s=1}^{\infty} R^{-s} E_t[x^a_{t+s}], \tag{RIV}$$

where x^a_t is abnormal earnings defined as $x^a_t \equiv x_t - (R-1)b_{t-1}$. RIV states that the value of the firm is equal to book value plus the discounted present value of expected future abnormal earnings (or residual income). The second term in RIV suggests that currently observable information will be relevant in valuation if it is relevant in forecasting future abnormal earnings.

In order to establish links between the value of the firm's equity and currently observable financial statement numbers, most related prior research has assumed an abnormal earnings dynamics (or forecasting) model and has then derived closed-form valuation expressions consistent with the assumed abnormal earnings dynamics. In contrast, we adopt a similar approach to Stark (1997) and assume a general, linear valuation model and then examine the implications for the abnormal earnings dynamics and other model properties. Our model is based on the following assumption:

A3. The value of the firm is a linear function of current period accounting items:[4]

$$P_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \beta_3 b_t + \beta_4 d_t. \tag{VAL1}$$

A4. Finally, we assume that dividends *per se* are irrelevant in valuation and that dividend payments reduce market value dollar-for-dollar, consistent with Miller and Modigliani (1961):

$$\partial P_t / \partial d_t = -1. \tag{MM}$$

This assumption is a *property* of the Ohlson (1995) model. However, given our modeling approach, it seems a reasonable assumption to introduce to provide further model structure.

Using VAL1 and MM it is straightforward to recast the general linear valuation expression as follows:

$$P_t = b_t + \alpha_1 x_t^a + \alpha_2 x_{2t} + \alpha_3 b_{t-1} \quad \text{(VAL2)}$$

where

$\alpha_1 = \beta_1 + \beta_4$,

$\alpha_2 = \beta_2 - \beta_1$, and

$\alpha_3 = \beta_1(R-1) + \beta_4 R$.

The first two terms in VAL2 closely resemble the abnormal earnings-based valuation expression (5) in Ohlson (1995) (ignoring "other information"). These terms indicate that the value of the firm is equal to book equity value plus a multiple of abnormal earnings. However, VAL2 also indicates that under our model setup, and in contrast to Ohlson (1995), the value of the firm also depends on earnings component x_{2t} and on lagged book value. Our subsequent analysis shows that these differences are associated with two accounting properties: whether earnings components aggregate in valuation and whether accounting is unbiased or conservative.

While the form of VAL2 shares similarities with the Ohlson (1995) model, the assumptions underpinning it are very different. If the set of current accounting items are sufficient for linear valuation using VAL1, then VAL2 follows using only clean surplus accounting and dividend irrelevance assumptions. Specifically it is unnecessary to assume no-arbitrage pricing (A1) or specific abnormal earnings dynamics. In contrast, Ohlson (1995) assumes no arbitrage and an abnormal earnings dynamics structure, along with clean surplus accounting. However, while our model setup does not make explicit assumptions about the abnormal earnings dynamics it does *imply* a general abnormal earnings dynamics structure, if one also introduces the no-arbitrage assumption, as stated in the following result:

Lemma 1 *Assume no arbitrage (A1), clean surplus accounting (A2), linear valuation (A3) and dividend irrelevance (A4). The following abnormal earnings dynamics are* *implied:*

$$E_t[x^a_{t+1}] = \omega_1 x^a_t + \omega_2(Rx_{2t} - E_t[x_{2t+1}]) + \omega_3(Rb_{t-1} - b_t) \qquad \textit{(ABED)}$$
$$= \omega_1 x^a_t + \omega_2(Rx_{2t} - E_t[x_{2t+1}]) + \omega_3(d_t - x^a_t)$$

where

$\omega_1 = R\alpha_1/(1 + \alpha_1)$,

$\omega_2 = \alpha_2/(1 + \alpha_1)$,

$\omega_3 = \alpha_3/(1 + \alpha_1)$.

Proof: See the appendix.

ABED clearly indicates that abnormal earnings dynamics parameters articulate with valuation parameters, as suggested by RIV. The first autoregressive term in the abnormal earnings dynamics ABED is similar to the abnormal earnings dynamics in the Ohlson (1995) model. However, the other two terms in ABED distinguish the abnormal earnings dynamics implied by our model from Ohlson (1995). The second term $\omega_2(Rx_{2t} - E_t[x_{2t+1}])$ will be important if the valuation weights on earnings components differ and if the expected rate of growth in next period's earnings component x_{2t} is different from $R - 1$. The third term is important if valuation parameter α_3 is different from zero and if the realized current period rate of growth in book value differs from $R - 1$. The second line of expression ABED exploits the clean surplus relation to replace $(Rb_{t-1} - b_t)$ by the difference between (net) dividends paid and abnormal earnings - book equity grows at a rate greater than (less than) $R - 1$ when $d_t < x^a_t$ ($d_t > x^a_t$). We show later that dependence of the abnormal earnings dynamics on growth is a property of conservative accounting.[5]

RIV utilizes only the no-arbitrage and clean surplus accounting assumptions and suggests that if book equity value is known, one need only forecast abnormal earnings realizations for all future periods in order to value the firm. The same must also be true for the abnormal earnings dynamics ABED, because it is consistent with the same two assumptions. However, introduction of the additional assumptions of linear valuation (VAL1) and dividend irrelevance (MM) implies the abnormal earnings forecasting model ABED. An interesting feature of the implied forecasting model is that if $\omega_2 \neq 0$ then earnings component x_{2t} has a role in forecasting abnormal earnings. A second interesting feature is that abnormal earnings expectations themselves depend on expectations of future realizations of earnings component x_{2t+1}, i.e. $E_t[x^a_{t+1}]$ depends on $E_t[x_{2t+1}]$. This in turn implies that the two earnings components are co-dependent.

The potential information role of earnings components is an important point of difference between our model and the prior literature. ABED suggests that if $\omega_2 > -1$ then the two earnings components are expected to be negatively associated, *ceteris paribus*. Specifically, $\partial E[x_{1t+1}]/\partial E[x_{2t+1}] = -(1 + \omega_2)$.[6] This implies restrictions on the information dynamics governing the individual earnings components. We return to this issue when we discuss the case of a valuation irrelevant earnings component.

2 Aggregation of earnings components

2.1 The relation between valuation and abnormal earnings dynamics

Earnings components aggregate in valuation when the valuation parameters on the two earnings components are identical, i.e. $\beta_1 = \beta_2 \Leftrightarrow \alpha_2 = 0$. Imposing this constraint on earnings components valuation parameters eliminates earnings components from both valuation and abnormal earnings dynamics expressions and leads to the following results:

Proposition 1: *If earnings components aggregate in valuation then:*

(i) Abnormal earnings-based valuation: *the value of the firm can be written in terms of book value and abnormal earnings as follows:*

$$P_t = b_t + \alpha_1 x^a_t + \alpha_3 b_{t-1} \qquad \text{(AGG1)}$$

where

$\alpha_1 = \beta_1 + \beta_4$, *and*

$\alpha_3 = \beta_1(R-1) + \beta_4 R$.

(ii) Weighted average valuation: *if* $\beta_4 \neq 0$, *the value of the firm can be expressed as a weighted average of book value and capitalized earnings, adjusted for dividends, as follows:*

$$P_t = (1-k)b_t + k(\varphi x_t - d_t) \tag{AGG2}$$

where

$k = (R-1)\alpha_1 - \alpha_3 = -\beta_4$ *and*

$\varphi = (R\alpha_1 - \alpha_3)/[(R-1)\alpha_1 - \alpha_3] = -\beta_1/\beta_4$.

(iii) Abnormal earnings dynamics: *the following abnormal earnings dynamics are implied*:

$$\begin{aligned} \widetilde{x}^a_{t+1} &= \omega_1 x^a_t + \omega_3(Rb_{t-1} - b_t) + \varepsilon_{t+1} \\ &= \omega_1 x^a_t + \omega_3(d_t - x^a_t) + \varepsilon_{t+1} \end{aligned} \tag{ABED1}$$

where

$\omega_1 = R\alpha_1/(\alpha_1 + 1)$,

$\omega_3 = \alpha_3/(\alpha_1 + 1)$ and

ε_{t+1} is a mean zero error term.

Further, assuming a mild regularity condition, AGG1 is also implied by ABED1.[7]

Proof: See the appendix.

Valuation expressions AGG1 and AGG2 are generalizations of the Ohlson (1995) valuation model. In expression AGG1, similar to Ohlson (1995), firm value is anchored on current book value, to which is added a multiple of current abnormal earnings. Although the model places no restrictions on the sign of the valuation parameter α_1, it is reasonable to assume for the purposes of subsequent analysis that *ceteris paribus* higher abnormal earnings lead to higher market value of equity, and thus that $\alpha_1 \geq 0$. AGG1 differs from Ohlson's abnormal earnings-based valuation expression because of the term $\alpha_3 b_{t-1}$. As shown below, this term is related to whether accounting is unbiased or conservative.

Valuation expression AGG2 expresses the value of the firm as a "weighted average" of book value and dividend-adjusted capitalized earnings and again resembles the weighted average valuation expression in Ohlson (1995). However, whereas in Ohlson (1995) the capitalization factor φ is constant and equal to $R/(R-1)$, in our model this is only the case if $\alpha_3 = 0$. Further the valuation weight k also differs from the analogous term in the Ohlson (1995) model except when $\alpha_3 = 0$. We show below that the differences in k and φ between Ohlson (1995) and our model result from corrections for accounting conservatism.

ABED1 identifies the abnormal earnings dynamics implied by our model when earnings components aggregate in valuation. It nests the abnormal earnings dynamics assumed in the Ohlson (1995) model (ignoring "other information" terms). In common with Ohlson's abnormal earnings dynamics, abnormal earnings depend on an autoregressive component and the term $\omega_1 x_t^a$ is directly analogous. However, the second term on the right hand side of ABED1, $\omega_3(Rb_{t-1} - b_t)$ (or equivalently $\omega_3(d_t - x_t^a)$), does not feature in Ohlson's abnormal earnings dynamics. This term captures the impact of growth in book value on future abnormal earnings and, as shown below, is related to the degree of accounting conservatism.

Expression ABED1 shows clearly how valuation parameters and abnormal earnings dynamics parameters articulate. The autoregressive parameter ω_1 is linked to the valuation parameter on abnormal earnings in AGG1 in a manner equivalent to Ohlson (1995), i.e. $\alpha_1 = \omega_1/(R - \omega_1)$. Thus α_1 increases with the persistence of abnormal earnings, and vice versa. As in Ohlson (1995), when abnormal earnings have no persistence ($\omega_1 = 0$) they are irrelevant in valuation ($\alpha_1 = 0$), and as ω_1 approaches unity, the valuation multiple on abnormal earnings approaches the capitalization factor $1/(R-1)$. Further, similar to the Ohlson (1995) model, in the weighted average valuation model the valuation weight k and the capitalization factor φ are related positively to abnormal earnings persistence. When $\alpha_3 = 0$ both parameters are identical to their counterparts in the Ohlson (1995) model.

ABED1 emphasizes the links between informational relevance in forecasting abnormal earnings and valuation relevance. It shows that if earnings components aggregate (are individually irrelevant) in valuation they also aggregate (are individually irrelevant) in forecasting abnormal earnings. As indicated in Proposition 1(iii), it is straightforward to show that the reverse is also true - if earnings components aggregate in forecasting abnormal earnings they also aggregate in valuation. It is worth noting that this property of the model can be inferred directly from RIV, and therefore is unrelated to the assumption of dividend irrelevance.

One further property of the information dynamics is particularly noteworthy. Expectations of individual earnings components are unnecessary in forecasting abnormal earnings because $\omega_2 = 0$ and this implies that holding x_t^a and $(Rb_{t-1} - b_t)$ constant, $\partial E[x_{1t+1}]/\partial E[x_{2t+1}] = -1$. Thus, when earnings components aggregate in valuation they are expected to trade-off against each other dollar-for-dollar at the margin. This implies that the parameters of the information dynamics governing the two earnings components are complementary and must aggregate in a manner consistent with ABED1.[8]

2.2 Unbiased accounting and conservative accounting

We now show how the valuation expressions AGG1 and AGG2 and abnormal earnings dynamics expression ABED1 capture the possibility that accounting is conservative. Ohlson (1995) defines accounting as unbiased if book value and market value are asymptotically equal, i.e. if the expected value of unrecorded goodwill $E_t[P_{t+s} - b_{t+s}] \rightarrow 0$, as $s \rightarrow \infty$. Accounting is conservative if market value is expected to exceed book value asymptotically, i.e. if $E_t[P_{t+s} - b_{t+s}] > 0$ as $s \rightarrow \infty$. RIV indicates that accounting will be conservative if $E_t[x^a_{t+s}] > 0$ as $t \to \infty$, for any $s > 0$. It can be shown that the valuation parameter α_3 (or equivalently abnormal earnings dynamics parameter ω_3) determines whether the accounting system is unbiased or conservative. The following proposition summarizes how accounting conservatism is captured in the valuation and abnormal earnings dynamics expressions:

Proposition 2: *Assume* $0 \leq \alpha_1 < \frac{1}{R-1} \Leftrightarrow 0 \leq \omega_1 < 1$ *and that book value satisfies the asymptotic property*: $\lim_{s\to\infty} E_t[b_{t+s}] > 0$. If *earnings components aggregate in valuation, then:*

(i) Abnormal earnings-based valuation: *unbiased accounting implies and is implied by* $\alpha_3 = 0$. *Accounting is conservative if and only if* $\alpha_3 > 0$. *The degree of accounting conservatism increases with* α_3.

(ii) Weighted average valuation: *unbiased accounting implies and is implied by the earnings capitalization factor* $\varphi = R/(R-1)$. *Accounting is conservative if and only if* $\varphi > R/(R-1)$. *As the degree of accounting conservatism increases, ceteris paribus* φ *increases and* k *decreases.*

(iii) Abnormal earnings dynamics: *unbiased accounting implies and is implied by* $\omega_3 = 0$. *Accounting is conservative if and only if* $\omega_3 > 0$.

Proof: See the appendix.

2.2.1 Valuation and conservative accounting

Proposition 2 shows that the valuation parameter α_3 encapsulates the biasedness property of the accounting system. Proposition 2 (i) indicates that the term $\alpha_3 b_{t-1}$ in AGG1 represents a valuation correction for accounting conservatism, where the degree of conservatism increases with the parameter α_3. The conservatism correction can be viewed as an adjustment to the time t book value "anchor" to which capitalized abnormal earnings is added to obtain firm value. Under an alternative perspective, the conservatism adjustment can be viewed as a reduction in the capital charge levied in computing abnormal earnings which when capitalized can be added to time t book value to obtain the value of the firm.[9] Under this perspective the effective capital charge is made at a rate of $(R - 1 - \alpha_3/\alpha_1) < R - 1$ per dollar of book value when accounting is conservative.[10]

Expression AGG2 also accommodates corrections for accounting conservatism. Proposition 2(ii) states that when accounting is unbiased the capitalization factor applied to earnings in the weighted average valuation model is identical to Ohlson (1995), i.e. $R/(R-1)$. However, when earnings recognition is conservative, earnings are capitalized at a strictly higher rate. Consistent with notions that higher quality (more conservative) earnings attract higher valuation multiples, the capitalization rate φ increases with the degree of accounting conservatism.

Conservatism also affects the weighting parameter k. Holding constant the valuation parameter α_1 (and therefore the degree of persistence in abnormal earnings), the effective weight attached to capitalized earnings decreases with accounting conservatism, and the effective weight on book value increases. The positive dependence between the book value weight and the degree of accounting conservatism compensates for understatement of book value under conservative accounting and makes intuitive sense. Since the weights on book value and capitalized earnings are complements, the weight on capitalized earnings decreases as conservatism increases.

Although we assume that the persistence parameter ω_1 lies between zero and unity, as in Ohlson (1995), the model places no restrictions on k, which in the Ohlson (1995) model is restricted to lie between zero and one and depends only on the persistence of abnormal earnings. In our model the weighting parameter k reflects a trade-off between abnormal earnings persistence and conservatism. In particular, if accounting is sufficiently conservative given the level of persistence in abnormal earnings such that $\alpha_1 < \alpha_3/(R-1)$, then k will be negative and the valuation coefficient on dividends β_4 $(= -k)$ will be positive.

This property of our model is interesting in light of the finding in empirical tests of the Ohlson model that the valuation coefficient on dividends is often positive, contrary to model predictions (see, e.g., Rees, 1997; Akbar and Stark, 2003; and Hand and Landsman, 2004). One possible explanation for this result is that dividends contain information signaling future abnormal earnings. Our model provides an alternative, accounting-based explanation for the positive pricing of dividends that is consistent with Modigliani–Miller dividend irrelevance.

2.2.2 Abnormal earnings dynamics and conservative accounting

The abnormal earnings dynamics parameters in ABED1 articulate with valuation parameters and reflect accounting conservatism. Specifically, the term $\omega_3(Rb_{t-1} - b_t)$ in the first ABED1 expression and the equivalent term $\omega_3(d_t - x_t^a)$ in the second expression reflect an adjustment to abnormal earnings expectations that allows for accounting conservatism. The parameter $\omega_3 = \alpha_3/(1+\alpha_1)$ is positive when accounting is conservative ($\alpha_3 > 0$). However the conservatism adjustment in abnormal earnings expectations also depends on the growth in book value, which in turn is a function dividend payout/retentions, as captured by the terms $(Rb_{t-1} - b_t) = (d_t - x_t^a)$.

When there is "excess" growth in book value, dividends paid is less than abnormal earnings and $(Rb_{t-1} - b_t)$ is negative. However, following Feltham and Ohlson (1995), the long-run expected growth rate should be less than $R - 1$ to eliminate growth paradoxes and therefore $E_t(Rb_{t+s-1} - b_{t+s}) = E_t(d_{t+s} - x_{t+s}^a) > 0$ $(s \geq 1)$. Thus, on average, the conservatism adjustment in abnormal earnings projections based on ABED1 is expected to be positive - past and current conservatism is associated with higher future abnormal earnings. However, in any specific period the effect of accounting conservatism on (abnormal) earnings expectations depends on the contemporaneous rate of growth of the firm, and this depends on the dividend payout relative to the level of abnormal earnings. Holding the abnormal earnings persistence parameter constant, when the dividend payout is higher (lower) than abnormal earnings, expected future abnormal earnings are higher (lower) than predicted by a simple autoregressive expectations model.

The interaction between accounting conservatism and growth in conditioning earnings expectations in our model is consistent with early research examining growth-dependent biases in accounting numbers (see, e.g. Beaver and Dukes, 1973; Stauffer, 1971). The property of our implied abnormal earnings dynamics contrasts with the dynamics assumed by Feltham and Ohlson (1995) where, if accounting is conservative, expected abnormal earnings depends on the *level* of book value (of operating assets) and this dependence is not a function of contemporaneous growth (although valuation parameters do depend on the long-run expected growth rate).

2.2.3 Dividend displacement and conservatism

Dependence of abnormal earnings dynamics on the rate of growth of the firm has implications for the displacement effect of dividends on future earnings, discussed by Ohlson (1995) and Penman and Sougiannis (1997). The dividend irrelevance assumption A4 states that the payment of dividends reduces firm value dollar-for-dollar. However, the effect of dividend payout on expected future earnings depends on the properties of accounting. It follows directly from ABED1 and CSR that when earnings components aggregate, the displacement effect of a dollar of dividends paid at t on earnings at $t+1$ is:

$$\partial E[x_{t+1}]/\partial d_t = -(R - 1 - \omega_3). \qquad \text{(DIV)}$$

The dividend displacement effect identified by Ohlson (1995), $\partial E[x_{t+1}]/\partial d_t = -(R - 1)$, is the special case of our model where accounting is unbiased and the conservatism parameter $\omega_3 = 0$. DIV indicates that when earnings components aggregate and accounting is conservative $\partial E[x_{t+1}]/\partial d_t > -(R-1)$.[11] In words, when accounting is conservative a dollar of dividends reduces one-period ahead expected earnings by less the cost of equity. Equivalently, conservative accounting causes the marginal accounting rate of return on a dollar of retained earnings to be less than the economic rate of return. Again, this property of our model has some intuitive appeal.

2.3 Earnings component aggregation: summary

The results presented in Proposition 1 extend the Ohlson (1995) valuation model to a more descriptive setting where accounting is conservative. As in the Ohlson model, abnormal earnings and book values are sufficient summary financial statement measures for valuation. Conservatism is captured by one parameter in the model, which in turn affects the valuation coefficients on earnings and dividends. The valuation expressions in Ohlson (1995) are modified by simple conservatism adjustments – the abnormal earnings-based valuation model also includes a multiple of lagged book value, while the weighted average valuation expression incorporates adjustments embedded in the earnings capitalization factor and the valuation weights. Our analysis further shows that when earnings components aggregate in valuation they also aggregate in forecasting abnormal earnings and that valuation parameters articulate with abnormal earnings dynamics parameters. The degree of accounting conservatism is captured by a single abnormal earnings dynamics parameter linked to growth in book value, or equivalently to the level of dividends relative to abnormal earnings. This conservatism parameter is also important in determining the degree to which a dollar of dividends displaces future earnings.

3 Valuation-irrelevant earnings components

3.1 Valuation and abnormal earnings dynamics

There are several reasons for believing that it is unrealistic to treat all components of clean surplus earnings as if they aggregate in valuation. Accounting practice in many countries permits dirty surplus accounting for items such as currency gains and losses and asset revaluations. Empirical estimates of valuation models often suggest that earnings components attract different valuation weights, contrary to Proposition 2. Further, analysts often focus on earnings constructs such as "core earnings" that exclude "unusual" clean surplus earnings components, e.g. "one-time" exceptional and extraordinary items resulting from charges, asset write-downs and provisions. Recognition of such items can be justified on the grounds of conservative accounting, but they are excluded because core earnings are believed to provide a better guide in forecasting future earnings and profitability. In this section we use our model to extend the "transitory earnings" analysis of Ohlson (1999). Specifically, we examine the properties that a component of earnings must satisfy if it is to be treated as a dirty surplus, valuation-irrelevant flow.

We follow Ohlson (1999) and examine the case where earnings component x_{2t} can be combined with contemporaneous dividends without loss of information for valuation. Formally, when x_{2t} is valuation irrelevant the accounting information set $Z_t^* = \{x_{1t}, b_t, (d_t - x_{2t})\}$ contains the same information as $Z_t = \{x_{1t}, x_{2t}, b_t, d_t\}$ for the purpose of valuation.[12] Under this irrelevance definition, the valuation coefficient on x_{2t} is equal and opposite to the valuation coefficient on dividends, i.e. $\beta_2 = -\beta_4$. The information set Z_t^* can be characterized as resulting from an accounting system where the income statement reports "core" earnings x_{1t}, earnings component x_{2t} is "deleted" from the income statement and treated as a dirty surplus flow, and the statement of retained earnings shows the net change in retained clean surplus earnings $(d_t - x_{2t})$.

Using similar analysis as for the aggregation case, we can obtain valuation and abnormal earnings dynamics expressions consistent with the valuation irrelevance of x_{2t} by setting $\beta_2 = -\beta_4$, as follows:

Proposition 3: *If x_{2t} is irrelevant in valuation then*:

(i) Core abnormal earnings-based valuation: *the value of the firm can be written in terms of book value and core abnormal earnings as follows:*

$$P_t = b_t + \alpha_1^* x_{1t}^a + \alpha_3^* b_{t-1} \tag{CORE1}$$

where

$x^a_{1t} \equiv x_{1t} - (R-1)b_{t-1}$ *is core abnormal earnings,*

$\alpha^*_1 = \beta_1 - \beta_2$ *and*

$\alpha^*_3 = \beta_1(R-1) - \beta_2 R.$

(ii) Weighted average valuation and core earnings: *if* $\beta_2, \beta_4 \neq 0$, *the value of the firm can be expressed as a weighted average of book value and capitalized core earnings, adjusted for dividends as follows:*

$$P_t = (1-k^*)b_t + k^*[\varphi^* x_{1t} - (d_t - x_{2t})] \tag{CORE2}$$

where

$k^* = \beta_2 = (R-1)\alpha^*_1 - \alpha^*_3$ *and*

$\varphi^* = \beta_1/\beta_2 = (R\alpha^*_1 - \alpha^*_3)/[(R-1)\alpha^*_1 - \alpha^*_3].$

(iii) Abnormal earnings dynamics and core earnings: *the abnormal earnings dynamics are as follows:*

$$E_t[x^a_{t+1}] = \omega^*_1 x^a_{1t} + \omega^*_1 E_t[x_{2t+1}]/R + \omega^*_3(Rb_{t-1} - b_t) \tag{ABED2}$$

where

$\omega^*_1 = R\alpha^*_1/(1+\alpha^*_1)$, *and*

$\omega^*_3 = \alpha^*_3/(1+\alpha^*_1).$

Further, assuming a mild regularity condition, CORE1 is implied by ABED2.

Proof: See the appendix.

Just as Proposition 1 nests the Ohlson (1995) valuation model, Proposition 3 nests the Ohlson (1999) transitory earnings valuation model. It shows that when earnings component x_2 is informationally irrelevant in valuation, one may replace aggregate earnings by core earnings and then apply a valuation expression that is identical in form to AGG1. Similar to the aggregation case, it is reasonable to assume that *ceteris* *paribus* higher core abnormal earnings leads to higher market value of equity and thus that $\alpha_1^* \geq 0$. Further, just as AGG1 differs from the analogous valuation expression in Ohlson (1995) because of the term $\alpha_3 b_{t-1}$, so too valuation expression CORE1 differs from the core abnormal earnings valuation expression in Ohlson (1999) because of the term $\alpha_3^* b_{t-1}$. We show later that this term can be interpreted as a valuation correction for accounting conservatism.

The symmetry between the aggregate earnings- and core earnings-based valuation functions extends to the weighted average valuation approach and CORE2 has identical informational requirements to Ohlson's (1999) transitory earnings model - only book value, core earnings and dividends net of the irrelevant earnings component are necessary for valuation. Note that similar to the aggregation case, the capitalization factor φ^* and the valuation weight k^* depend on α_3^*. This, in turn, depends on accounting conservatism, discussed further below.

The abnormal earnings dynamics ABED2 shares common features with ABED1, but with aggregate earnings being replaced by core earnings. The parameters ω_1^* and ω_3^* articulate with valuation parameters α_1^* and α_3^* in exactly the same ways as their counterparts in the aggregation case - $0 \leq \omega_1^* < 1$ implies and is implied by $0 \leq \alpha_1^* < \frac{1}{R-1}$. Further, there is equivalence between valuation-irrelevance of the lagged book value term in CORE1 and irrelevance of the term $(Rb_{t-1} - b_t)$ in predicting abnormal earnings, i.e. $\alpha_3^* = 0$ implies and is implied by $\omega_3^* = 0$. However, the main difference between ABED2 and the aggregation case is the presence of the term $\omega_1^* E_t[x_{2t+1}]/R$. ABED2 suggests that x_{2t+1} can be forecastable, even though it is irrelevant in valuation. If x_{2t+1} is both irrelevant and predictable, then for the purposes of forecasting abnormal earnings we can combine the discounted expected value of x_{2t+1} with core abnormal earnings in period t. We return to the implications of the predictability of x_2 below. But we note at this stage that the Ohlson (1999) information dynamics are a special case of ABED2.

3.2 Conservative accounting

When the irrelevant earnings component is unpredictable, it is also possible to show that the biasedness properties of the dirty surplus accounting system are symmetric with the aggregation case. Results are summarized in the following proposition:

Proposition 4: *Assume that* $0 \leq \alpha_1^* < \frac{1}{R-1} \Leftrightarrow 0 \leq \omega_1^* < 1$ *and book value satisfies the asymptotic property*: $\lim_{s\to\infty} E_t[b_{t+s}] > 0$. *If earnings component* x_{2t} *is irrelevant in valuation and unpredictable then:*

(i) CORE1: *Unbiased accounting implies and is implied by* $\alpha_3^* = 0$. *Accounting is conservative if and only if* $\alpha_3^* > 0$. *The degree of accounting conservatism increases with* α_3^*.

(ii) CORE2: *Unbiased accounting implies and is implied by the earnings capitalization factor* $\varphi^* = R/(R-1)$. *Accounting is conservative if and only if* $\varphi^* > R/(R-1)$. *As the degree of accounting conservatism increases, ceteris paribus* φ^* *increases and* k^* *decreases.*

(iii) ABED2: *Unbiased accounting implies and is implied by* $\omega_3^* = 0$. *Accounting is conservative if and only if* $\omega_3^* > 0$.[13]

Proof: See the appendix.

Thus for the special case where the dirty surplus component is unpredictable, i.e., $E_t[x_{2t+1}] = 0$, the valuation parameter α_3^* plays an identical role in adjusting for accounting bias as does α_3 in the aggregation case. It acts as an adjustment to the book value anchor in the abnormal earnings-based valuation expression CORE1. Equivalently, it can be interpreted as a conservatism-related reduction to the capital charge levied in calculating core abnormal earnings. The degree of accounting conservatism increases with α_3^* (and equivalently increases with ω_3^*). Further, adjustments for accounting conservatism in the valuation weights and the capitalization factor in the weighted average valuation expression, CORE2, depend on α_3^* and are exactly analogous to the aggregation case.

3.3 Dynamics of earnings components

Proposition 3 assumes dividend irrelevance, $\partial P_t/\partial d_t = -1$. Since the valuation coefficient of a dollar of earnings component x_2 is equal and opposite to a dollar of dividends, it follows that $\partial P_t/\partial x_{2t} = 1$. This is also an attribute of transitory earnings in the Ohlson (1999) model. Valuation expressions CORE1 and CORE2 are consistent with this property because x_{2t} increases book value dollar for dollar as a result of CSR. Reconciling the properties $\partial P_t/\partial x_{2t} = 1$ and $\partial b_t/\partial x_{2t} = 1$ with RIV makes clear that the present value of future abnormal earnings must not depend on x_{2t}. But what are the dynamic properties of the irrelevant earnings component implied by our model? Similar to Ohlson (1999) we examine, in turn, two characteristics of the irrelevant component - predictability and relevance in forecasting.

First, consider the predictability of x_{2t+1}. The most important point of difference between ABED2 and the information dynamics assumed by Ohlson (1999) is the dependence of abnormal earnings expectations on $E_t[x_{2t+1}]$. This results from interdependence between $E_t[x_{1t+1}]$ and $E_t[x_{2t+1}]$. Rearrangement of ABED2 indicates that if x_{2t} is valuation irrelevant *and* predictable, expected values of the two components must be related as follows:

$$E_t[x_{1t+1}] + E_t[x_{2t+1}]/(1 + \alpha_1^*) = Y_t \qquad \text{(DSED)}$$

where $Y_t = (R - 1)b_t + \omega_1^* x_{1t}^a + \omega_3^*(Rb_{t-1} - b_t)$. Given the information set at time t and hence Y_t, the two earnings components are substitutes, but there is not a dollar-for-dollar trade-off between them. Rather $(1 + \alpha_1^*)$ dollars of x_{2t+1} trades off against one dollar of x_{1t+1}. This expected "rate of exchange" between the two earnings components reflects the fact that in CORE1 a dollar of core (abnormal) earnings component x_1 has a valuation coefficient of α_1^*, while a dollar of earnings component

x_2 only affects firm value because it affects book value. The value of the firm at $t+1$ is unaffected if a dollar of x_{1t+1} is replaced by $(1+\alpha_1^*)$ dollars of x_{2t+1}.

DSED does not require that x_{2t+1} is unpredictable. For example, if $E_t[x_{2t+1}] = \omega_{22}x_{2t}$, DSED will hold as long as $E_t[x_{1t+1}] = Y_t - \omega_{22}x_{2t}/(1+\alpha_1^*)$. Thus x_{2t} can be valuation-irrelevant and yet still have a role in forecasting individual accounting items. Generally if $E_t[x_{2t+1}]$ depends on x_{2t} then $E_t[x_{1t+1}]$ must also depend on x_{2t}, but the marginal effects of x_{2t} on $E_t[x_{1t+1}]$ and $E_t[x_{2t+1}]$ will "cancel out" so that abnormal earnings expectations do not depend on x_{2t}. Conversely, if x_{2t} is not relevant in forecasting x_{2t+1}, then x_{2t} must also be irrelevant in forecasting x_{1t+1} since the marginal effects of x_{2t} on both $E_t[x_{1t+1}]$ and $E_t[x_{2t+1}]$ must be zero.

To summarize, our linear valuation model can accommodate an earnings component that is irrelevant in valuation in exactly the same way as transitory earnings in the Ohlson (1999) model. And yet the valuation-irrelevant earnings component is not necessarily "transitory", as defined by Ohlson. x_{2t+1} can be predictable and x_{2t} can have a role in forecasting individual accounting items, and yet x_{2t} can still be valuation-irrelevant. But is this difference between our model and the Ohlson (1999) model a purely technical result arising from different model assumptions? Or does it allow us to extend further the analysis of transactions giving rise to valuation-irrelevant earnings components?

3.4 Implications

The finding that x_2 can be accounted for as a value-irrelevant earnings component even though it is predictable provides a justification for dirty surplus accounting in cases other than purely transitory earnings components. ABED2 can be interpreted reflecting a "mixed" accounting model combining traditional realization-based accounting, giving rise to the persistent earnings component x_1, and fair value accounting, leading to x_2. Expression DSED characterizes the effects of choosing between the two accounting methods. Realization-based accounting produces a stream of core abnormal earnings in future periods. This stream is predictable as a consequence of the autoregressive term in ABED2. Valuation expression CORE1 indicates that core abnormal earnings attracts a valuation coefficient equal to α_1^*. In contrast, fair value accounting through x_2 brings forward recognition of the α_1^* dollars of value. If DSED holds then fair value accounting through x_2 reduces earnings component x_1 in current and future periods. The present value of the changes in x_1 resulting from the substitution of x_{2t} is α_1^* dollars.

The trade-off captured by Proposition 3 between immediate recognition and gradual realization of an earnings component is fundamental to many important accounting policy debates. One example is the treatment of asset write-downs, restructuring charges and other related provisions. Such charges are often economically large and predictable. One approach is to invoke the conservatism principle to immediately write-off expected future charges, classifying the write-off as "extraordinary". Consistent with the dirty surplus treatment of such flows, analysts often focus on earnings pre-extraordinary items in forecasting earnings and in estimating intrinsic value. An alternative approach is to gradually match such charges against other earnings components. Proposition 3 suggests that financial statements can be equally informative under the two alternative accounting treatments. Core earnings-based valuation expressions CORE1 and CORE2 can be applied if an immediate write-off that equals to the fair value of the projected charges is classified as a dirty surplus earnings component x_2 and if the write-off is irrelevant for predicting future abnormal earnings. Alternatively, the charges can be recognized and valued as a component of core earnings, with an immediate expense of $1/(1+\alpha_1^*)$ dollar for every dollar of immediate write-off. Further expense recognition will follow in future periods as core abnormal earnings evolves according to DSED. Although it can be argued that financial statement integrity is enhanced by reducing management's ability to "inventory" future profits through write-offs and provisions, our results confirm that in the absence of opportunistic management behavior, such write-offs will not necessarily reduce the informativeness of financial statements, even if they are predictable.

4 Summary and conclusions

In this paper we analyze an accounting-based valuation model comprising four financial statement items - two earnings components, book value and dividends. Assuming linear valuation, no-arbitrage, dividend irrelevance and clean surplus accounting, we show that when earnings components aggregate, valuation expressions and abnormal earnings dynamics are generalizations of the Ohlson (1995) model, incorporating simple adjustments for accounting conservatism. In particular, if earnings components aggregate and accounting is conservative, abnormal earnings-based valuation involves adding a multiple of lagged book value to the Ohlson (1995) value estimate. This is equivalent to reducing the capital charge applied in calculating abnormal earnings when accounting is conservative. Under the weighted average valuation approach, when earnings components aggregate the valuation parameters can be adjusted to allow for conservatism. Both the valuation multiple on book value and the earnings capitalization factor increase with accounting conservatism. Under both valuation approaches the conservatism adjustments are closely linked to a single abnormal earnings dynamics parameter. In turn, the abnormal earnings dynamics depends on two lagged book value terms.

We also analyze accounting systems where a "core" earnings component is the relevant earnings construct for valuation and the second earnings component is irrelevant and treated as a dirty surplus accounting flow that may be netted off against dividends without loss of information for valuation. Valuation expressions closely resemble the aggregation case, but core (abnormal) earnings replaces clean surplus (abnormal) earnings. We demonstrate that an earnings component can be irrelevant in valuation even when it is predictable and even if it is relevant in forecasting other accounting items. However, although knowledge of the earnings component is unnecessary for valuation, its dynamics may be relevant because earnings components may be co-dependent. This suggests that a subtle but important dimension of a dirty surplus accounting system will be whether it reveals sufficient information concerning the information dynamics to enable identification of relevant valuation parameters.

Our dirty surplus accounting results are potentially useful in interpreting accounting policy debates reflecting tension between application of the conservatism principle and a desire to match earnings components to produce a "smooth" pattern of earnings realizations. Examples include write-offs and charges associated with restructuring, amortization of goodwill versus direct write-off to reserves and the treatment of asset revaluation surpluses. Our analysis can justify dirty surplus treatment of accounting flows from a valuation perspective, as long as there is substitution between core earnings and the dirty surplus component at a rate that depends on the relevant valuation coefficients. However, we note that information asymmetries and resulting agency problems play no role in our modeling approach. Such considerations might well be important in resolving accounting policy controversies in practice.

Appendix

Proof of Lemma:

Valuation expression VAL1 and the clean surplus relation CSR give:

$$P_t = (\beta_3 - \beta_4)b_t + (\beta_1 + \beta_4)x_t^a + (\beta_2 - \beta_1)x_{2t} + (\beta_1(R-1) + \beta_4 R)b_{t-1}$$

The mathematical restrictions in A2 and dividend irrelevance A3 together imply:

$$\beta_4 = \beta_3 - 1. \tag{MM1}$$

Given MM1, the first expression simplifies to the following general valuation expression:

$$P_t = b_t + \alpha_1 x_t^a + \alpha_2 x_{2t} + \alpha_3 b_{t-1} \tag{VAL2}$$

where $\alpha_1 = \beta_1 + \beta_4$, $\alpha_2 = \beta_2 - \beta_1$, and $\alpha_3 = \beta_1(R-1) + \beta_4 R$.

Using the definition of abnormal earnings, CSR can be rewritten as:

$$b_{t+1} = x_{t+1}^a - d_{t+1} + Rb_t. \tag{CSR2}$$

Define unrecorded goodwill g_t such that $P_t \equiv b_t + g_t$. The no-arbitrage condition A4 can then be restated as:

$$E_t[b_{t+1} + g_{t+1} + d_{t+1}] = R(b_t + g_t).$$

Substituting for b_{t+1} from CSR gives:

$$E_t[g_{t+1} + x_{t+1}^a] = Rg_t. \tag{GW}$$

From VAL2, $g_t = \alpha_1 x_t^a + \alpha_2 x_{2t} + \alpha_3 b_{t-1}$. Substituting for g_{t+1} and g_t in GW we obtain the following general abnormal earnings dynamics:

$$\begin{aligned} E_t[\tilde{x}_{t+1}^a] &= \omega_1 x_t^a + \omega_2(Rx_{2t} - E_t[x_{2t+1}]) + \omega_3(Rb_{t-1} - b_t) \\ &= \omega_1 x_t^a + \omega_2(Rx_{2t} - E_t[x_{2t+1}]) + \omega_3(d_t - x_t^a) \end{aligned}$$

where $\omega_1 = R\alpha_1/(1+\alpha_1)$, $\omega_2 = \alpha_2/(1+\alpha_1)$, $\omega_3 = \alpha_3/(1+\alpha_1)$. Q.E.D.

Proof of Proposition 1:

(i) Aggregation (i.e. $\beta_2 = \beta_1$) implies that $\alpha_2 = 0$ and hence VAL2 reduces to AGG1.

(ii) Noting CSR, it is straightforward to rewrite AGG1 as AGG2 if $\alpha_3 \neq (R-1)\alpha_1$ or $\beta_4 \neq 0$. If $\alpha_3 = (R-1)\alpha_1$, then $\beta_3 = 1$ and $P_t = b_t + \beta_1 x_t$.

(iii) Aggregation implies that $\omega_2 = 0$. ABED hence reduces to ABED1. The second expression follows by noting CSR.

Applying CSR2 and VAL2 to the no-arbitrage condition A1 gives

$$E_t[(1+\alpha_1)x^a_{t+1} + \alpha_2 x_{2t+1} + (R+\alpha_3)b_t] = R[b_t + \alpha_1 x^a_t + \alpha_2 x_{2t} + \alpha_3 b_{t-1}].$$

Assume ABED1 and $E_t[\widetilde{x}_{2t+1}] = \omega_{21}x^a_t + \omega_{22}x_{2t} + \omega_{23}b_t + \omega_{24}b_{t-1}$, then by matching the coefficients we have $\alpha_2\omega_{22} = R\alpha_2$. Under a mild regularity condition, say, $\omega_{22} < R$, it must follow that $\alpha_2 = 0$. Thus, ABED1 implies AGG1. Q.E.D.

Proof of Proposition 2:

By applying AGG1 and ABED1:

$$\begin{aligned} E_t[P_{t+s} - b_{t+s}] &= \alpha_1 E_t[x^a_{t+s}] + \alpha_3 E_t[b_{t+s-1}] \\ &= \alpha_1\omega_1 E_t[x^a_{t+s-1}] + \frac{\omega_3}{R}\left\{E_t[b_{t+s-1}] + R\alpha_1 E_t[b_{t+s-2}]\right\} \end{aligned}$$

By induction,

$$E_t[P_{t+s} - b_{t+s}] = \alpha_1\omega_1^s x^a_t + \frac{\omega_3}{R}\left\{\sum_{j=1}^{s}\omega_1^{j-1}E_t[b_{t+s-j}] + R\alpha_1\omega_1^{s-1}b_{t-1}\right\}.$$

With the assumption $0 \leq \omega_1 < 1$, it is clear that accounting is unbiased if and only if $\omega_3 = 0$ or equivalently if $\alpha_3 = (1+\alpha_1)\omega_3/R = 0$. Hence $\varphi = (R\alpha_1 - \alpha_3)/((R-1)\alpha_1 - \alpha_3) = R/(R-1)$.

Accounting is conservative if and only if

$$\lim_{s\to\infty} E[P_{t+s} - b_{t+s}] = \frac{\omega_3}{R}\lim_{s\to\infty}\sum_{j=1}^{s}\omega_1^{j-1}E_t[b_{t+s-j}] > 0$$

This holds if and only if $\omega_3 > 0 \Leftrightarrow \alpha_3 > 0$. The above expression shows that $\lim_{s\to\infty} E[P_{t+s} - b_{t+s}]$ is a positive function of α_3. It follows that the degree of accounting conservatism increases with α_3. Since $\partial\varphi/\partial\alpha_3 > 0$, φ increases with α_3 and k decreases with α_3. Therefore, accounting is conservative if and only if $\varphi > R/(R-1)$. As the degree of accounting conservatism increases, φ increases and k decreases. Q.E.D.

Proof of Proposition 3:

(i) Assuming dirty surplus accounting (i.e. $\beta_1 \neq \beta_2 = -\beta_4$), VAL2 implies

$$P_t = b_t + \alpha_1^* x_t^a - \alpha_1^* x_{2t} + \alpha_3^* b_{t-1}$$

where $\alpha_1^* = \beta_1 - \beta_2$ and $\alpha_3^* = (R-1)\alpha_1^* - \beta_2$. Since $x_t^a = x_{1t}^a + x_{2t}$, the above implies CORE1.

(ii) CORE2 follows directly from CORE1 and CSR.

(iii) $\alpha_1 + \alpha_2 = 0$ implies $\omega_1 + R\omega_2 = 0$. Reorganizing ABED, we have ABED2.

Applying CSR2 and VAL2 to no-arbitrage condition A1 gives

$$E_t[(1+\alpha_1)x_{t+1}^a + \alpha_2 x_{2t+1} + (R+\alpha_3)b_t] = R[b_t + \alpha_1 x_t^a + \alpha_2 x_{2t} + \alpha_3 b_{t-1}]$$

If ABED2 holds, then

$$\begin{aligned} & [(1+\alpha_1)\omega_1^* - R\alpha_1]x_{1t}^a + [(1+\alpha_1)\omega_1^*/R + \alpha_2]E_t[\widetilde{x}_{2t+1}] - R(\alpha_1+\alpha_2)x_{2t} \\ = \; & R[\alpha_3 - (1+\alpha_1)\omega_3^*]b_{t-1} - [\alpha_3 - (1+\alpha_1)\omega_3^*]b_t \end{aligned}$$

Assume $E_t[x_{2t+1}] = \omega_{21}^* x_{1t}^a + \omega_{22}^* x_{2t} + \omega_{23}^* b_t + \omega_{24}^* b_{t-1}$. By matching the coefficients of x_{1t}^a and x_{2t}, we have

$$\begin{aligned} (1+\alpha_1)\omega_1^* - R\alpha_1 + [(1+\alpha_1)\omega_1^*/R + \alpha_2]\omega_{21}^* &= 0 \\ [(1+\alpha_1)\omega_1^*/R + \alpha_2]\omega_{22}^* - R(\alpha_1+\alpha_2) &= 0 \end{aligned}$$

Which implies $(R\alpha_2 + (1+\alpha_1)\omega_1^*)(\omega_{21}^* + R - \omega_{22}^*) = 0$. Under a mild regularity condition, say, $\omega_{22}^* < \omega_{21}^* + R$, it must follow that $R\alpha_2 + (1+\alpha_1)\omega_1^* = 0$ or $(\beta_1 - \beta_2)(\beta_4 + \beta_2) = 0$. Since $\beta_1 \neq \beta_2$, so $\beta_4 + \beta_2 = 0$. That is, CORE1 is implied by ABED2. Q.E.D.

Proof of Proposition 4:

Note that $g_t = \alpha_1^* x_{1t}^a + \alpha_3^* b_{t-1}$ and $\omega_3^* = \alpha_3^*/(1+\alpha_1^*)$. Since $E_t[x_{2t+1}] = 0$, ABED2 implies $E_t[x_{1t+1}^a] = \omega_1^* x_{1t}^a - \omega_3^*(b_t - Rb_{t-1})$. By induction, we have

$$E_t[x_{1t+s}^a] = \omega_1^{*^s} x_{1t}^a + \omega_3^* \sum_{j=0}^{s-1} \omega_1^{*^j} \left\{E_t[Rb_{t+s-2-j} - b_{t+s-1-j}]\right\}$$

Analysis of unbiased (conservative) accounting is essentially the same as in the aggregation case. Q.E.D.

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Notes

1. Ohlson (2003) argues that it is possible to reparameterize the model Ohlson (1995) model to be consistent with conservative accounting by assuming that abnormal earnings mean revert to a positive constant.

2. Not all results depend on all four assumptions. Necessary assumptions are indicated at the relevant points in the analysis.

3. In the case of earnings component aggregation, we need only impose the weaker restriction $\partial(x_{1t} + x_{2t})/\partial d_t = 0$ in place of $\partial x_{1t}/\partial d_t = 0$ and $\partial x_{2t}/\partial d_t = 0$.

4. VAL1 is consistent with Markovian linear information dynamics (LID) with the form $Z_{t+1} = \Omega Z_t + \varepsilon'_{t+1}$, where ε'_{t+1} is a vector of noise terms with each element mean zero and uncorrelated with elements of Z_t. See Stark (1997) and Pope and Wang (2004) for further analysis. The regularity condition for this model is that the maximum characteristic root of the constant matrix Ω is less than R. Note also that we could allow for additional "other information" variables capturing information that affects market values before it is recognized in the accounting system as in Ohlson (1995). The inclusion of other information terms will be innocuous for the main results of the paper as long as they are contemporaneously uncorrelated with the other variables included in the model, as in Ohlson (1995, 1999).

5. Feltham and Ohlson (1995) distinguish between (net) financial assets and operating assets. Dividends are paid from financial assets and operating assets are assumed to be independent of dividends. They then assume abnormal (operating) earnings dynamics based on abnormal *operating* earnings and book value of *operating* assets.

They show that when accounting is conservative the abnormal (operating) earnings dynamics coefficient on operating assets is positive. The inconsistency between the Feltham and Ohlson (1995) model and ours arises because of the independence of operating assets from dividends. As a consequence of this, there is no constraint in the Feltham and Ohlson (1995) model on the valuation coefficient on book value (of operating assets) relative to the valuation coefficient on dividends. In contrast, in our model VAL1 and dividend irrelevance implies $\beta_4 = \beta_3 - 1$. This in turn implies that $(Rb_{t-1} - b_t)$ has a coefficient ω_3 in our abnormal earnings dynamics.

6. Specifically, $E_t[x_{1t+1}] = (R-1)b_t + \omega_1 x_t^a + \omega_2 R x_{2t} - (\omega_2+1)E_t[x_{2t+1}] + \omega_3(Rb_{t-1} - b_t)$.

7. Assume $E_t[\tilde{x}_{2t+1}] = \omega_{21}x_t^a + \omega_{22}x_{2t} + \omega_{23}b_t + \omega_{24}b_{t-1}$. The condition $\omega_{22} < R$ is sufficient.

8. For example, if the linear information dynamics governing earnings component i is assumed to be $x_{it+1} = \omega_{i1}x_{1t}^a + \omega_{i2}x_{2t} + \omega_{i3}b_t + \omega_{i4}b_{t-1} + \varepsilon_{it}$ $(i = 1, 2)$ then aggregation requires that $\omega_{11} + \omega_{21} = \omega_1$, $\omega_{12} + \omega_{22} = 0$, $\omega_{13} + \omega_{23} = -\omega_3$ and $\omega_{14} + \omega_{24} = \omega_3 R$.

9. AGG1 may be rewritten as $P_t = b_t + \alpha_1 x_t^{a^*}$ where $x_t^{a^*} \equiv x_t - (R - 1 - \alpha_3/\alpha_1)b_{t-1}$.

10. Also, similar to Feltham and Ohlson (1995), for the model to be interesting, it is necessary for the model to apply to a going concern therefore to rule out long-run liquidation resulting from a negative expected long-run growth rate.

11. Ohlson (1995) also discusses the two-period ahead displacement effect of dividends on earnings. In our model, the displacement effect of dividends on two-period aggregate earnings, adjusted for the displacement effect of expected dividends at $t+1$, can be shown to be:

$$\partial E[x_{t+2} + x_{t+1} + d_{t+1}(R - 1 - \omega_3)]/\partial d_t = -(R^2 - 1) + (R + \omega_1 - \omega_3)\omega_3$$

In comparison, in the Ohlson (1995) unbiased accounting model the two-period ahead displacement effect is equal to $-(R^2-1)$. Again, when earnings components aggregate in valuation and $\omega_3 = 0$, the two models are equivalent.

12. As noted in Ohlson (1999), there are two other potentially interesting combinations of accounting items: $\{x_{1t}, (b_{t-1} + x_{2t}), d_t\}$ and $\{x_{1t}, (b_t - x_{2t}), d_t\}$. We do not

consider these cases in the present paper.

13. When dirty surplus accounting applies $(\beta_2 = -\beta_4)$, from CSR and ABED2 the one-period ahead displacement effect of dividends on earnings can be shown to be:

$$\partial E[x_{t+1}]/\partial d_t = -(R - 1 - \omega_3^*) + (\omega_1^*/R)\partial E_t[x_{2t+1}]/\partial d_t$$

The bias-related parameter ω_3^* and the one-period ahead displacement effect of dividends on earnings component x_{2t} in the abnormal earnings dynamics differentiate our model from the Ohlson (1999) model and from the aggregate earnings case above. Consistent with Proposition 4, conservative accounting implies and is implied by $\partial E[\widetilde{x}_{t+1}]/\partial d_t > -(R - 1) + (\omega_1^*/R)\partial E_t[\widetilde{x}_{2t+1}]/\partial d_t$.

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