

Correlation Effects in Credit Risk Models with Incomplete Accounting Information*

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Abstract

In this paper the credit risk model of Duffie and Lando (2001) is extended to cases where the true asset value process and the noise term are correlated. I find theoretical and empirical evidence for the importance of the inclusion of the correlation. In particular, I find empirical evidence for a negative correlation. Under the assumption that the true asset value and the noise term are correlated, I derive a corporate bond pricing formula in nearly closed form. It is then demonstrated that introducing the correlation has a statistical and an information effect. For small accounting noise the statistical effect is dominant, whereas for large accounting noise the information effect is dominant. Ignoring the correlation yields significant pricing errors in corporate bond prices.

JEL: G12, G13, G33.

Keywords: Credit Risk Models, Incomplete Accounting Information, Correlations.

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1 Introduction

In this paper, I incorporate strategic behavior by the management into credit risk models where investors have only incomplete information about the value of the assets of the company. Credit risk models are often divided into structural models and reduced form models. In structural models an asset value process is specified. Default occurs when the value of the assets is lower than a given boundary, e.g. the nominal value of the liabilities. The first structural models were Black and Scholes (1973) and Merton (1974). In these models it is assumed that equity is a call option on the assets of a company with a strike price equal to the nominal value of debt. As a consequence, the price of corporate debt is the value of the assets minus the value of the call option or alternatively the value of a risk free bond minus a put option written on the assets. While in these early models default is only possible at maturity, Black and Cox (1976) allow for default prior to maturity. In their model default occurs when an asset value process hits the default boundary for the first time. Structural models that incorporate assumptions about the optimal capital structure are Leland (1994) and Leland and Toft (1996). Longstaff and Schwartz (1995) incorporate a stochastic interest rate into the structural model framework. Besides their role in academia, structural models have also proven to be successful in the banking industry. The company KMV provides a structural credit risk model to calculate the expected default frequency. A description of the KMV model is given in Bohn and Crosby (2003). Although structural models are a good description of an economic situation, they fail to explain the observed credit spreads in bond and credit derivative markets. Since the asset value process is usually specified as a geometric Brownian motion, credit spreads should be close to zero for short times to maturity. But this is not the case. Reduced form models avoid this problem by modeling default as the first jump of a pure jump process, typically a Poisson process. Default intensities and recovery rates are assigned exogenously to the model. It is possible to calibrate reduced form models to market prices of debt. In contrast to structural models, default is not predictable. Models which study reduced form models are Artzner and Delbaen (1995), Jarrow, Lando and Turnbull (1997), Duffie and Singleton (1999), and Lando (1998). Reduced form models do not specify an asset value process and do not use accounting based information. That is the reason why they are criticized for lacking economic content.

On first view, structural models and reduced form models seem to be very different. This is not necessarily the case. Duffie and Lando (2001) are the first to show that structural models can be transferred into reduced form models by reducing the information set of the market participants. In their model, they assume that the investors cannot observe the true asset value process, but instead receive a noisy signal about the asset value, for example an accounting report. Duffie and Lando (2001) show that with the reduced information set, a structural model can admit an intensity, hence default is unpredictable. Giesecke (2005) extends this idea to the case where the default barrier is unobservable, Zhou (2001) adds an unpredictable jump time to the asset value process, and Collin-Dufresne, Goldstein and Helwege (2002) assume that investors receive lagged information about the asset value process. Baglioni and Cherubini (2005) distinguish between noisy accounting information resulting from estimation errors and noisy accounting information resulting from fraud. Further models that assume incomplete information about the asset value process are Giesecke and Goldberg (2004), Cetin, Jarrow, Protter and Yildirim (2004), and Guo, Jarrow and Zeng (2005).

In this paper, the focus will be on credit risk models with an incomplete information structure as given in Duffie and Lando (2001). In their model, it is assumed that the noisy accounting signal consists of the true asset value and a normally distributed noise term. Furthermore, it is assumed that the true asset value process and the noise process are uncorrelated. But in practice the two processes are not necessarily uncorrelated, because the management of the company acts opportunistically. Numerous studies in the accounting literature give evidence that the management of a company has incentives to do earnings management. Many of these studies come to the conclusion that companies prefer to report smooth asset values through time. Smoothed asset values mean that in the model of Duffie and Lando (2001) the true asset value term and the noise term are negatively correlated. Other studies come to the conclusion that the management reacts to bonus schemes which can yield in a positive correlation between the true asset value term and the noise term.

The correlation is estimated for a data set consisting of accounting reports for German companies for the years 1998–2003. Under the assumption that the market value of the company is the true asset value and the asset value given in the financial statements is

the noisy reported asset value, I derive a series of noise terms. I find that the correlation between the logarithm of the true asset value and the noise term is -0.178 for the whole sample, while for the sub-sample of *Neuer Markt* companies it is -0.672 .

With the theoretical and empirical arguments at hand, I model the opportunistic behavior of the management by a correlation between the logarithm of the true asset value process and the noise process in the model of Duffie and Lando (2001). This is done by assuming that the two processes are bivariate normally distributed with a correlation coefficient ρ . Under this assumption, I derive a nearly closed form formula for the pricing of corporate bonds. I then show that, for usual parametrization, the correlation between the logarithm of the asset value and the noise term is very similar to the correlation of the true asset value and the noise term.

Moreover, I study the effects of the correlation coefficient on credit spreads of corporate zero bonds. I find that a correlation introduces two pricing effects: It (1) has a statistical effect, and it (2) has an information effect. The statistical effect arises from the new distribution of the reported asset values. For example for a negative correlation extreme asset values become less likely. The information effect says that if the investor observes a certain reported asset value, he rationally infers the asset value by taking the correlation into account. For example, if the investor observes an extreme reported asset value and the correlation is negative, the true asset value might be even more extreme. Whether the first or the second effect is dominant, depends on parameters like the standard deviation of the noise process (accounting noise) and the level of the reported asset value. I show that for small accounting noise, the statistical effect of the correlation is dominant while for medium and large accounting noise the information effect is dominant.

Furthermore, I ask the question, how large the pricing error is if the correlation is neglected. Assuming correlations scenarios of -0.178 and -0.672 , as estimated in the empirical analysis, I find pricing errors for corporate zero bonds with 5 years to maturity between 2 and 10 basis points for the first scenario and between 16 and 33 basis points for the second scenario.

The rest of the paper is organized as follows: Chapter 2 gives a brief review of the basic model of Duffie and Lando (2001). In Chapter 3, I argue from a theoretical and an

empirical point of view that a correlation between the noise term and the true asset value process has to be included in the basic model of Duffie and Lando (2001). In Chapter 4, I derive the conditional default probability and risky bond prices for the Duffie and Lando (2001) model enhanced with a correlation. The implications of the enhanced model are studied in Chapter 5, while Chapter 6 concludes.

2 The Basic Model of Duffie and Lando (2001)

In this section, I briefly review the model of Duffie and Lando (2001). In the model, it is assumed that the assets of a company follow a geometric Brownian motion. A default occurs when the assets are worth less than a predefined boundary. While the assumptions so far are standard in structural models, Duffie and Lando (2001) specify that investors can only observe a noisy signal \widehat{V}_t of the asset value, e.g. an accounting report, instead of the true asset value V_t . They further assume that the logarithm of the noisy signal can be decomposed into the logarithm of the true asset value and into a normally distributed noise term U_t . The logarithm of the noisy signal can be written as

$$\ln \widehat{V}_t = \ln V_t + U_t. \quad (1)$$

In Equation (1) it is assumed that $\ln V_t$ and $U(t)$ are independent. In this setup, Duffie and Lando (2001) derive the default probability of the company conditional on the noisy accounting signal and on the information whether the company is already in the default state or not. With the default probability derived under reduced information, it is possible to explain stylized facts observed in the corporate bond markets. For example, even for very small times to maturity there is still a positive default probability and hence a strictly positive credit spread. Duffie and Lando (2001) prove that — assuming the reduced information set — the default time is completely unpredictable. Therefore, they show that modeling default risk with incomplete information about the asset value can link structural models, that are based on an economic concept, to reduced form models, where default comes as a surprise.

The implications of the unobservability of the true asset values on corporate bond prices is demonstrated in Figure 1. We see that introducing accounting noise bounds the credit spreads away from zero for very short maturities. Duffie and Lando (2001) interpret the noise term U as accounting noise. One can see in Figure 1 that the effect of the accounting noise, measured by its standard deviation, depends on the parametrization of the model. While economical considerations would predict that a higher accounting noise increases the default probability and thus credit spreads (see first graph), one can

see situations where a higher level of accounting noise decreases the default probability and hence credit spreads (see second graph). To understand this anomaly, one has to keep in mind that the model of Duffie and Lando (2001) does not make any assumptions about strategic behavior of the management. Hence, we see a pure statistical effect. When the reported asset value is very low then a high accounting noise is better for the investor because the chance that the true asset value is above the reported asset value is higher than the chance that the true asset value is lower than the reported asset value. In contrast, if the accounting noise is small the investor knows for sure that the company is in a bad situation. The model of Duffie and Lando (2001) ignores possible interactions between accounting noise and true asset value models.

3 Theoretical and Empirical Motivation for a Correlation

3.1 Theoretical Motivation for a Correlation

In the basic model of Duffie and Lando (2001) it is assumed that the logarithm of the true asset value process and the noise process are uncorrelated. The expected value of the noise term can be interpreted as a reporting bias regardless of the actual value of the company. Usually, one would expect a downward bias because almost all accounting regimes report rather prudently. The standard deviation of the noise term can be interpreted as accounting noise. Accounting noise can arise even when the management of the company does not have perfect information when it assesses the asset value. As a consequence the management has to rely on estimations. And even if the management had perfect information about the asset values, it might be forced to report in accordance to accounting rules that introduce noise. Neither of the two noise parameters, expected value and standard deviation, does account for strategic behavior of the management. By allowing for correlation, it is possible to make the noise process dependent on the true asset value process. A negative correlation between the true asset value term and the noise term can be interpreted as a smoothing of reported asset values. In states in which the true asset value is relatively high, firms tend to report asset values lower than the true asset values. In states of relatively low asset values, firms tend to report higher asset values than the true ones. Alternatively, a positive correlation has the interpretation that the management prefers high fluctuations in reported asset values.

Managers have different incentives to misreport the true asset value. Baglioni and Cherubini (2005) argue that managers have incentives to hide bad asset values and to understate performance in good states. Reasons for overstating the performance in bad states are (1) that a low asset value might have the consequence of an intervention by shareholders or debt holders, (2) that the good reputation of the manager can be damaged, and (3) that the compensation of the manager is linked to market values or reported asset values. In good states, reasons to understate the asset values are (1) the reduction of the tax burden, (2) to hide profits to benefit personally, and (3) a strategic default.

The discussion of the management of reported asset values is related to the discussion of earnings management. Under the assumption that a company does not change its

financing, earnings for a certain period can be calculated as today's reported asset value minus the reported asset value of the previous period. Hence, there is a direct link between reported earnings and reported asset values. In the accounting literature, there are two competing hypotheses regarding earnings management. The first hypothesis says that the management tries to smooth earnings. Models that support this hypothesis are Fudenberg and Tirole (1995), DeFond and Park (1997), and Trueman and Titman (1988). All studies conclude that the management has incentives to smooth reported earnings. Although, all studies come to similar results, they assume different reasons in their models. Trueman and Titman (1988) argue that the management wants to lower the costs of capital, whereas DeFond and Park (1997) and Trueman and Titman (1988) argue that managers are led by the goal to keep their job. According to the earnings smoothing hypothesis the correlation between the logarithm of the true asset value and the noise term should be negative.

The second hypothesis mainly goes back to Healy (1985). He argues that the management observes the proceedings from operations and then chooses discretionary accruals in a way to maximize benefits from bonuses. In years where the proceedings from operations are either significantly higher or significantly lower than the bonus boundary, the management chooses negative discretionary accruals. In all other states the management chooses positive discretionary accruals. According to the hypothesis of Healy (1985), it is not clear which sign a correlation between the logarithm of the true asset value process and the noise term would have. Closely related to the second hypothesis is the so called *big bath* accounting. In a bad year the company's income statements are manipulated to look even worse. As a consequence the earnings in the following years are artificially high. Taking a *big bath* can result in a positive correlation between the logarithm of the true asset value process and the noise term.

Although there is still a discussion in the accounting literature whether and to what extent firms smooth reported earnings, the theoretical arguments provide evidence to control for correlation in credit risk models with incomplete accounting information.

3.2 Empirical Estimations of the Correlation

In this subsection, I estimate the correlation between the logarithm of the true asset values and the noise term for a sample of public traded German companies. The purpose of the estimation procedure is to further motivate that the independence assumption between the logarithm of the true asset values and the noise term cannot be assumed in practice and hence the model of Duffie and Lando (2001) needs a modification.

3.2.1 Data

I use data from the Compustat (Global) Database for German companies of all industries for the years 1998–2003. I use only German companies to keep country-specific influences small. Restricting the analysis to German companies leaves me with 3,035 company years of observation. Out of this 3,035 company years, 890 are from companies traded on the *Neuer Markt*, a special trading segment on the Frankfurt Stock Exchange for growth-oriented and innovative companies. A particularity of the German accounting rules is that since 1998, companies are allowed to prepare their financial statements in accordance to either German GAAP, US GAAP, or IAS. Using the accounting regime as a selection criterion, the sample can be subdivided into 1,639 company years of German GAAP reports, 866 company years of IAS reports, and 530 company years of US GAAP reports.

3.2.2 Correlation Estimation

The estimation of the correlation between the logarithm of the true asset value process and the noise term is not easy. Initially, one would want to infer the accounting noise parameters and the correlation coefficient from bond data. Unfortunately, this approach does not work properly. The main problem is that in most cases bond data is only available for companies whose equity is also listed. In such a case it cannot be assumed that the asset value process is unknown to the investor. Following Herkommer (2006), I employ a different approach. I estimate the noise process from equity data.

It is known from the basic setup of Duffie and Lando (2001) that the observed asset

value process \widehat{V} can be decomposed into

$$\ln \widehat{V}_t = \ln V_t + U_t, \quad (2)$$

where V_t is the true asset value and U_t is the noise term. To obtain values for U_t , I interpret the book value of the assets (BV_i) of the sample firms as \widehat{V} . The corresponding equity values 2.5 months¹ ahead of the book values plus the book values of the liabilities (MV_i) are interpreted as V_t . Under these assumptions, one can calculate a sample of noise terms

$$U_i = \ln BV_i - \ln MV_i. \quad (3)$$

For the derivation of the series for the noise term U_i , I need the assumption of a *naive* investor. The investor is naive in the way that he uses the book value of the assets as the only accounting signal. He then rationally infers the noise parameters from the calculated series U_i , instead of using all available information such as other balance sheet positions or ratios. A key assumption of this procedure is that the noise parameters that are derived for public companies are also valid for private companies for which no *true* market values are available. For further discussion of the assumption of a *naive* investor, I refer to Herkommer (2006).

The correlation between U and $\ln V$ is estimated by the empirical correlation between the series of logarithms of market values $\ln MV_i$ and the calculated U_i series.

3.2.3 Results

The estimated correlation between the logarithm of the true asset value process and the noise term depends on the chosen sub-sample. For the whole sample, I calculate a correlation of -0.178 . The negative correlation corresponds to the idea that companies tend to smooth their reported asset values. Although, the correlation value is rather small in absolute terms, the standard error of the correlation is very small (0.018) so that the correlation is significantly different from zero for all usual significance levels.

¹KPMG (2002) finds out that German companies need on average 2.5 months from the end of the accounting year until they publish their financial reports.

When looking at sub-samples, one gets further evidence for the importance of the correlation between the logarithm of the true asset value and the noise term. In a first step, I divide the sample into a sub-sample of companies that were traded on *Neuer Markt* and into a sub-sample of companies that were traded in other markets. The estimated correlation for the companies traded on *Neuer Markt* is -0.672 with a standard error of 0.025 . Estimating the correlation for companies that were traded on other markets yields a correlation of -0.098 with a standard error of 0.021 . The results for the sub-sample of *Neuer Markt* companies demonstrate that the absolute size of the correlation can be large. It seems that *Neuer Markt* companies make stronger use of smoothing in their accounting reports than companies trading on other markets. This might stem from the fact that there is more asymmetric information involved in the relationship between growth companies and their financiers than for more established companies.

Furthermore, I investigate if the estimated correlation depends on the accounting regime the company uses. Therefore, I estimated the correlation for sub-samples of companies reporting according to IAS, US GAAP, and German GAAP. The estimated correlation values are -0.195 (standard error: 0.033) for IAS companies, -0.221 (standard error: 0.042) for US GAAP companies, and -0.151 (standard error: 0.024) for German GAAP companies. The estimated correlations are very similar. The less negative correlation for German GAAP companies results from the fact that no German GAAP companies are traded on *Neuer Markt*. It seems that different accounting regimes have no large influence on the correlation between the true asset value and the noise term of a company.

In conclusion the data supports the idea that there is a non-zero correlation between the logarithm of the true asset value and the noise term of a company. I estimated negative correlations for all sub-samples. This corresponds to the idea that companies have incentives to smooth their reported asset values. The absolute magnitude of the correlation differs for several sub-samples. Especially for *Neuer Markt* companies the magnitude of the correlation is significantly different from zero. Hence, the assumption of independence between logarithms of true asset values and noise terms — as stated in Duffie and Lando (2001) — is not supported in this empirical study.

4 The Model of Duffie and Lando (2001) with Correlation

In this section, I extend the model of Duffie and Lando (2001) by allowing the noise term U to be correlated with the logarithm of the true asset value process.

4.1 Setup

I assume a fixed probability space (Ω, \mathcal{F}, P) on which an asset value process is defined that follows a geometric Brownian motion. Under the risk neutral probability measure the asset value process is given by

$$dV_t = \mu V_t dt + \sigma V_t d\widehat{W}_t, \quad (4)$$

where \widehat{W}_t is a standard Wiener process under the risk neutral measure. The parameters μ and σ stand for the drift and the diffusion of a geometric Brownian motion. Like in the setup of the basic model of Duffie and Lando (2001), default is modeled as the first time τ that the process hits the default barrier V_b . The investor cannot observe the true asset value V but instead receives noisy accounting reports \widehat{V} at discrete points of time t_1, \dots, t_n and the information $I_{\{\tau \leq s\}}$ if the company is already in default or not. Hence, his information is modeled by the filtration $(\mathcal{H}_t)_{t>0}$ generated by

$$\mathcal{H}_t = \sigma \left(\widehat{V}(t_1), \dots, \widehat{V}(t_n), I_{\{\tau \leq s\}} : 0 \leq s \leq t \right). \quad (5)$$

The information of the investor is different from $(\mathcal{F}_t)_{t>0}$ the filtration generated by

$$\mathcal{F}_t = \mathcal{H}_t \vee \sigma(V(t_1), \dots, V(t_n)). \quad (6)$$

As in Duffie and Lando (2001), I assume that the noisy asset value \widehat{V}_t can be decomposed into

$$Y_t = \ln \widehat{V}_t = Z_t + U_t, \quad (7)$$

where $Z_t = \ln(V_t)$, and — in contrast to Duffie and Lando (2001) — Z_t and U_t are bivariate normally distributed. The main objective is to derive the conditional density of V_t given the reduced information set \mathcal{H}_t . I restrict the setup to the case where only one noisy asset value report is observed at time t .² To derive the desired density, I need in a first step the density $b(x|Y_t, z_0, t)$ of Z_t stopped at $\tau = \inf\{t : Z_t \leq V_b\}$ for a fixed $Z_0 = z_0$, conditional on the observation of Y_t . Assume that Z_t and U_t are bivariate normally distributed with mean $\xi = (mt + z_0, \bar{u})$ and covariance matrix

$$\Sigma = \begin{pmatrix} \sigma^2 t & a\sigma\sqrt{t}\rho \\ a\sigma\sqrt{t}\rho & a^2 \end{pmatrix}, \quad (8)$$

where $m = \mu - 0.5\sigma^2$.

Then the density $b(x|Y_t, z_0, t)$ of Z_t stopped at $\tau = \inf\{t : Z_t \leq V_b\}$, conditional on the observation Y_t is given as

$$b(x|Y_t, z_0, t) = \frac{\psi(z_0 - v_b, x - v_b, \sigma\sqrt{t})\phi_{UZ}(Y_t - x, x)}{\phi_Y(Y_t)}, \quad (9)$$

where $\psi(z_0 - v_b, x - v_b, \sigma\sqrt{t})$ stands for the probability that $\min\{Z_s : 0 \leq s \leq t\} > 0$ given that Z starts on level $z_0 - v_b$ at time 0 and ends at level $x - v_b$ at time t . This probability can be written as

$$\psi(z_0 - v_b, x - v_b, \sigma\sqrt{t}) = 1 - \exp\left(-\frac{2(z_0 - v_b)(x - v_b)}{\sigma^2 t}\right). \quad (10)$$

The term $\phi_{UZ}(Y_t - x, x)$ stands for the density of the bivariate normal distribution for U and Z , and $\phi_Y(Y_t)$ stands for the density of the observed noisy asset value Y_t , where Y_t is normally distributed with mean $mt + z_0 + \bar{u}$ and variance $a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho$. In Proposition 1 I give an analytical expression for $b(x|Y_t, z_0, t)$.

Proposition 1. *The density $b(x|Y_t, z_0, t)$ of Z_t stopped at $\tau = \inf\{t : Z_t \leq V_b\}$, conditional on the observation Y_t can analytically be written as*

²An extension to multiple observation times is given in Duffie and Lando (2001).

$$b(x|Y_t, z_0, t) = \sqrt{\frac{\gamma_0}{\pi}} e^{-J(\tilde{y}, \tilde{x}, \tilde{z}_0)} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} \left(1 - e^{-\frac{2\tilde{z}_0\tilde{x}}{\sigma^2 t}}\right),$$

with

$$\begin{aligned} J(\tilde{y}, \tilde{x}, \tilde{z}_0) &= \frac{1}{2} \frac{1}{(1-\rho^2)} \left\{ \frac{(\tilde{y} - \tilde{x})^2}{a^2} + \frac{(\tilde{z}_0 + mt - \tilde{x})^2}{\sigma^2 t} - 2\rho \frac{(\tilde{y} - \tilde{x})(\tilde{z}_0 + mt - \tilde{x})}{a\sigma\sqrt{t}} \right\} \\ \gamma_0 &= \frac{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}{2a^2\sigma^2 t(1-\rho^2)} \\ \gamma_1 &= \frac{(\tilde{y} - (mt + \tilde{z}_0))^2}{a^2\sigma^2 t(1-\rho^2)}. \end{aligned}$$

The variables \tilde{y} , \tilde{x} , and \tilde{z}_0 stand for $y - v_b - \bar{u}$, $x - v_b$, and $z_0 - v_b$ respectively.

Proof. A proof for Proposition 1 is straight forward. Inserting the definition of $\phi_{UZ}(Y_t - x, x)$, $\phi_Y(Y_t)$, and $\psi(z_0 - v_b, x - v_b, \sigma\sqrt{t})$ into Equation (9) results after some algebra in Proposition 1. \square

It is known from Duffie and Lando (2001) that the density of the true asset value process $g(x|y, z_0, t)$ conditional on the noisy information Y and on $\tau > t$ can be calculated as $b(x|y, z_0, t)$ divided by the conditional survival probability $Q(\tau > t|Y_t)$

$$g(x|y, z_0, t) = \frac{b(x|y, z_0, t)}{Q(\tau > t|Y_t)}. \quad (11)$$

To evaluate Equation (11) analytically, one has to solve for $Q(\tau > t|Y_t)$ which can be written as

$$Q(\tau > t|Y_t) = \int_{v_b}^{\infty} b(z|y, z_0, t) dz. \quad (12)$$

Proposition 2 gives the analytical solution of $Q(\tau > t|Y_t)$.

Proposition 2. *The survival probability $Q(\tau > t|Y_t)$ conditioned on the noisy accounting information Y_t can be written as*

$$\begin{aligned} Q(\tau > t|Y_t) &= \int_{v_b}^{\infty} b(z|y, z_0, t) dz \\ &= \gamma_6 e^{\frac{1}{4}\frac{\gamma_1}{\gamma_0}} \left(\Phi(\gamma_4) e^{-\frac{1}{2}\gamma_2} - \Phi(\gamma_5) e^{-\frac{1}{2}\gamma_3} \right) \end{aligned}$$

with

$$\begin{aligned}
\gamma_0 &= \frac{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}{2a^2\sigma^2 t(1 - \rho^2)} \\
\gamma_1 &= \frac{(\tilde{y} - (mt + \tilde{z}_0))^2}{a^2\sigma^2 t(1 - \rho^2)} \\
\gamma_2 &= \frac{(\tilde{y} - (mt + \tilde{z}_0))^2}{\sigma^2 t + a^2 - 2a\sigma\sqrt{t}\rho} \\
\gamma_3 &= \frac{(\tilde{y} - (mt + \tilde{z}_0))^2 + 4\tilde{y}\tilde{z}_0}{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho} + 4a^2 \left(\frac{a^2\rho^2\tilde{z}_0^2 + a^2\tilde{z}_0mt - a\sigma\sqrt{t}\rho(\tilde{z}_0^2 + \tilde{z}_0mt + \tilde{y}\tilde{z}_0)}{(\sigma^2 t + a^2 - 2a\sigma\sqrt{t}\rho)a^2\sigma^2 t} \right) \\
\gamma_4 &= \frac{\tilde{y}\sigma^2 t + (mt + \tilde{z}_0)a^2 - (\tilde{y} + mt + \tilde{z}_0)a\sigma\sqrt{t}\rho}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}\sqrt{a^2\sigma^2 t(1 - \rho^2)}} \\
\gamma_5 &= \frac{\tilde{y}(\sigma^2 t - a\sigma\sqrt{t}\rho) + \tilde{z}_0(a^2(2\rho^2 - 1) - a\sigma\sqrt{t}\rho) + mt(a^2 - a\sigma\sqrt{t}\rho)}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}\sqrt{a^2\sigma^2 t(1 - \rho^2)}} \\
\gamma_6 &= \frac{\sqrt{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}},
\end{aligned}$$

where \tilde{y} and \tilde{z}_0 stand for $y - v_b - \bar{u}$ and $z_0 - v_b$ respectively. $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal variable.

Proof. The Proof is given in Appendix A. □

The density $g(x|y, z_0, t)$ is the integral building block to derive the risk neutral default probability $Q(\tau < T|\mathcal{H}_t)$ conditional on the reduced information set \mathcal{H}_t . It is known that the risk neutral default probability equals 1 minus the survival probability. Hence, $Q(\tau < T|\mathcal{H}_t)$ can be written as

$$Q(\tau < T|\mathcal{H}_t) = 1 - \int_{v_b}^{\infty} Q(\tau \geq T|\mathcal{F}_t)g(x|Y_t, z_0, t)dx. \quad (13)$$

Note that $Q(\tau \geq T|\mathcal{F}_t)$ is the survival probability under the information set \mathcal{F}_t that contains the true asset values. The probability $Q(\tau \geq T|\mathcal{F}_t)$ has to be weighed for every x with the density $g(x|Y_t, z_0, t)$ to obtain the survival probability under the reduced information set \mathcal{H}_t . The risk neutral survival probability given the information set \mathcal{F}_t is well known³ and can be written as

³See e.g. Bielecki and Rutkowski (2002), Harrison (1990).

$$Q(\tau \geq T | \mathcal{F}_t) = \Phi(d_1) - \exp\left(\frac{-2\tilde{x}m}{\sigma^2}\right) \Phi(d_2) \quad (14)$$

with

$$d_1 = \frac{\tilde{x} + m(T - t)}{\sigma\sqrt{T - t}}$$

and

$$d_2 = \frac{-\tilde{x} + m(T - t)}{\sigma\sqrt{T - t}}.$$

The risk neutral default probability $Q(\tau < T | \mathcal{H}_t)$ is needed in the bond pricing model described in the following subsection.

4.2 Risky Bond Prices

In this subsection, I will derive a nearly⁴ closed form pricing formula for corporate bonds. Thus, I consider a corporate bond promising the bondholders a face value F at time T . Without loss of generality, I assume further that the bond does not pay any coupon and the recovery rate ω is a constant fraction of the discounted face value of the bond.⁵ Using the stated assumptions, the price of a corporate zero bond $P_{t,T}$ is given by

$$P_{t,T} = B_t(T)F[1 - Q(\tau < T | \mathcal{H}_t)] + \omega B_t(T)FQ(\tau < T | \mathcal{H}_t), \quad (15)$$

where $B_t(T)$ is the price of a risk free treasury bond and $Q(\tau < T | \mathcal{H}_t)$ is given in Equation (13). If a fixed recovery rate of the face value is assumed, the pricing formula becomes more complicated. I refer to Herkommer (2006) for calculation details under the recovery of face value assumption.

⁴The pricing formula will be closed form up to the numerical integral in Equation (13).

⁵The assumption concerning the recovery rate is called the recovery of treasury model (RT-model). This particular assumption is often used in credit risk models, see e.g. Jarrow and Turnbull (1995).

4.3 Credit Spreads

Credit Spreads are measured by the difference between the yield of treasury bonds and the yield of corporate bonds that are identical in all aspects except the credit quality. I calculate the credit spread CS as

$$CS = -\frac{1}{T-t} \ln \left(\frac{P_{t,T}}{F} \right) - r, \quad (16)$$

where $T-t$ stands for the time to maturity, $P_{t,T}$ stands for the price of a corporate bond, F stands for the face value of the corporate bond, and r stands for the risk free rate or the yield of a treasury bond.

4.4 Correlation between U and V

Up to now, I have introduced a correlation between the logarithm of the true asset value and the noise term. In this subsection, I will show how this correlation relates to the correlation between the true asset value and the noise term. As $\ln(V)$ and U are bivariate normally distributed, I know from Stein's Lemma that

$$\text{cov}(V, U) = \text{cov}(\ln V, U)E[V]. \quad (17)$$

Inserting the well known expressions for $\text{cov}(\ln V, U)$ and $E[V]$, Equation (17) simplifies to

$$\text{cov}(V, U) = a\sigma\sqrt{t}\rho(\ln V, U)e^{z_0+mt+0.5\sigma^2}. \quad (18)$$

Having found an expression for the covariance between V and U , it is now easy to calculate the correlation $\rho(V, U)$ between V and U

$$\begin{aligned} \rho(V, U) &= \frac{\text{cov}(V, U)}{\sqrt{\text{var}(V)}\sqrt{\text{var}(U)}} \\ &= \rho(\ln V, U) \frac{\sigma\sqrt{t}e^{0.5\sigma^2t}}{\sqrt{e^{\sigma^2t}(e^{\sigma^2t}-1)}}. \end{aligned} \quad (19)$$

One can see in Equation (19) that the difference between the correlation $\rho(V, U)$ and the correlation $\rho(\ln V, U)$ depends only on σ and t . For usual parametrizations the two correlations are very similar. In Figure 3, I have plotted the relation between the correlation $\rho(V, U)$ and the correlation $\rho(\ln V, U)$ for volatility values σ of 0.01, 0.2, and 0.5. One can see that only for a volatility of 0.5 there is a notable difference between the two correlations.

5 Correlation Effects

5.1 Correlation Effects and Expected Reported Asset Values

Introducing a correlation coefficient between the logarithm of the true asset value and the noise term has a direct impact on the expectation of the reported asset value. This effect is important because the expectation of the reported asset value process is a reference point for the assessment whether the observed reported asset value is relatively high or relatively low. In the case of zero correlation, the expectation of the reported asset value \widehat{V}_t can be written as

$$\begin{aligned} E[\widehat{V}_t] &= E[V_t]E[e^{U_t}] \\ &= e^{z_0+mt+0.5\sigma^2t+\bar{u}+0.5a^2}. \end{aligned} \tag{20}$$

When the correlation is different from zero, the expectation of the reported asset value changes to

$$\begin{aligned} E[\widehat{V}_t] &= E[V_t]E[e^{U_t}] + \text{cov}(V_t, e^{U_t}) \\ &= e^{z_0+mt+0.5\sigma^2t+\bar{u}+0.5a^2+a\sigma\sqrt{t}\rho}. \end{aligned} \tag{21}$$

The difference between Equation (20) and Equation (21) is the term $a\sigma\sqrt{t}\rho$ in the exponent. It is important to be aware of this difference, especially when the investor knows that the correlation is nonzero. When one analyzes the effect of introducing a correlation one has to adjust other parameters such as \bar{u} or a to make $E[V_t]$ equal to $E[\widehat{V}_t]$. This can e.g. be done by setting \bar{u} to

$$\bar{u} = -0.5a^2 - a\sigma\sqrt{t}\rho. \tag{22}$$

With this mean adjustment one compensates for a bias that would otherwise be introduced.

5.2 Correlation Effects and Accounting Noise

In this subsection, I analyze the effect of introducing the correlation ρ between the logarithm of the true asset value process and the noise term for different levels of accounting noise a .

5.2.1 Base Case Parameters

The model of Duffie and Lando (2001) depends on several parameters. The base case parameters are essentially taken from Herkommer (2006). Table 1 summarizes the parameter choices. The logarithm of the true asset value process has a risk neutral drift m of 0.07 and a diffusion σ of 0.15. The true asset value of the pre-period V_0 is set to 86.3, the default barrier is set to 60.0. As a basic security, I assume a corporate zero bond with a time to maturity of 10 years and a face value F of 100. The interest rate r and the recovery rate ω are constants taking values of 0.04 and 0.5 respectively. The expectation of the noise process \bar{u} is set according to Equation (22) to guarantee that expected reported asset values are equal to expected asset values. In such a setup, the investor anticipates that the correlation also has an effect on the reported asset values. Therefore the investor has the same reference point as in the case with zero correlation. The underlying assumption of such a mean adjustment is that the investor knows that the true asset values and the noise terms are correlated.

5.2.2 Statistical and Information Effect

The correlation between the logarithm of the true asset value and the accounting noise does have an impact on the pricing of corporate bonds. The pricing impact stems from two sources. The first source results from pure statistical effects. The correlation changes the distribution of the noisy accounting report \hat{V} . Even if I adjust for the parameter \bar{u} , as proposed in the previous subsection, the distribution of \hat{V} changes. E.g. for a positive correlation, very high and very low reported asset values become more probable. Besides this pure statistical effect, the correlation also introduces what can be called an information effect. When the investor observes a noisy accounting report, and he knows that the noise

term and the true asset value are correlated, he draws conclusions from this fact for his personal assessment of the default probability. E.g. when he observes an extremely high or an extremely low reported asset value and he knows that the correlation is negative, he infers that the true asset value is even more extreme. Usually, the statistical effect and the information effect have different signs. Which of these two effects is dominant, depends on other parameters such as the noise parameter a and the reported asset value \widehat{V} .

5.2.3 Numerical Example

The effect of the interaction between accounting noise and logarithm of the true asset value on credit spreads depends on the level of the reported asset values relative to the expected asset values given the parameters and the start value of the asset value process. In this setup, the expected asset value — as well as the expected reported asset value — is 93.6. I do the analysis for three different reported asset values. In the first scenario the accounting report announces a high asset value of 120, in the second scenario the accounting report announces a medium asset value of 93.6, and in the third scenario the accounting report announces a low asset value of 65.

Scenario 1: High Reported Asset Value ($\widehat{V} = 120$)

In Figure 4, I plot a graph that demonstrates the role of the correlation ρ and the accounting noise a on credit spreads in the case of a high reported asset value of $\widehat{V} = 120$. One can see that for small accounting noise the credit spread decreases with increasing correlation. For medium and large accounting noise the credit spread increases with increasing correlation. This particular behavior can be explained by the statistical and the information effects described in the previous subsection. The reported asset value of 120 seems very high for the investor in comparison to the expected asset value of 93.6. For a small noise the statistical effect is dominant. When the correlation is high, the probability of extremely high reported asset values increases. Thus, the credit spread decreases with a higher correlation. For medium and large accounting noise the information effect dominates. When the correlation is high, the investor anticipates that the high reported asset value results from the correlation. Thus, the credit spread increases with a higher correlation.

Scenario 2: Medium Reported Asset Value ($\widehat{V} = 93.6$)

The graph for Scenario 2 is plotted in Figure 5. One can see that for small accounting noise the credit spread is decreasing with higher correlation. For medium and large accounting noise the credit spread is increasing with increasing correlation. This can be explained as follows. The reported asset value is exactly the expectation of the unobserved asset value process. For small accounting noise the statistical effect dominates. For high positive correlations the probability of extreme reported asset values increases. Because one knows that the firm is not in default, the probability of extremely high reported asset values is higher than the probability of extremely low reported asset values. Hence, credit spreads decrease with higher correlations. For medium and large accounting noise the information effect dominates. For small correlations the investor anticipates that the medium reported asset value arises from the negative correlation, and thus from income smoothing. Because he knows that the company has not defaulted so far, the probability that the true asset value is larger than the reported asset value is higher than the probability that the true asset value is smaller than the reported asset value. Therefore, credit spreads are decreasing for low correlations.

Scenario 3: Low Reported Asset Value ($\hat{V} = 65$)

The graph for Scenario 3 is plotted in Figure 6. As one can see, for small accounting noise credit spreads increase as the correlation increases. For medium and large accounting noise credit spreads decrease as the correlation increases. As in Scenario 1 and 2, the behavior of the credit spreads can be explained by the statistical and by the information effect. For small accounting noise the statistical effect dominates: higher correlations increase the probability of extremely high and extremely low reported asset values, thus the low reported asset value becomes more probable. For medium and large accounting noise the information effect dominates. When the investor receives a very low reported asset value and the correlation is high, he anticipates that this low asset value might arise from the fact that the company reports a lower asset value than the true asset value.

In conclusion, for small accounting noise the statistical effect dominates in all three scenarios. One explanation for this behavior is that the leeway for earnings management is very low if the accounting noise is low. For higher accounting noise, the management of the company has more possibilities for earnings management, and the earnings management has a greater impact. Therefore, for medium and high accounting noise the information

effect is dominant.

5.3 Ignoring the Correlation: How Large is the Pricing Error?

In this subsection, I assume an investor who ignores that there is a correlation between the logarithm of the true asset value and the noise term. I try to quantify the pricing error the investor would make by ignoring the correlation. This is different from the situation in the previous subsection, because neglecting the correlation also introduces a different expected reported asset value so that the pricing error depends on the correlation effect described in the previous subsection and on the change in the expected reported asset value. I assume further that the investor has information about the asset value parameters as given in Table 1. Moreover, he knows the noise parameters \bar{u} and a . For the calculations, I assume that the expectation of the noise process \bar{u} is -0.272 and the standard deviation of the noise process a equals 0.66 .⁶ The pricing error is calculated for two correlation scenarios. In the first scenario the correlation between $\ln(V)$ and U is set to -0.178 , in the second scenario the correlation is set to -0.672 . The two scenarios are based on my calculations in Section 2. The first scenario corresponds to the case where I calculated the correlation using the whole sample of German companies, the second scenario corresponds to the case where I calculated the correlation using a sub-sample consisting only of *Neuer Markt* companies. For both scenarios, I again assume three different reported asset values of 120, 93.6, and 65.

The calculated pricing errors are plotted in Figure 7. For all reported asset values and for both correlation scenarios, the credit spreads differ from the base case where a correlation of 0 is assumed. The pricing error for an investor who ignores that the logarithm of the true asset value is correlated with the noise term, is particularly high for short times to maturity. For a reported asset value of 120 the pricing error for a bond maturing in five years is 10 basis points in Scenario 1 and 33 basis points in Scenario 2. For a reported asset value of 93.6 the pricing error for a bond with five years to maturity is as large as 7 basis points in Scenario 1 and 28 basis points in Scenario 2. For a reported asset value of 65 the pricing errors for the same bond are 2 basis points for Scenario 1 and

⁶These are the values calculated by Herkommer (2006) for IAS reports.

16 basis points for Scenario 2. For longer times to maturities the pricing errors decrease.

The calculated pricing errors demonstrate that ignoring the correlation between the true asset value and the noise term yields significant pricing errors, especially if the time to maturity is short.

6 Conclusion

In this paper, the model of Duffie and Lando (2001) was extended to the case where the logarithm of the true asset value and the noise term in reported asset values are bivariate normally distributed with a correlation different from zero. I provided a theoretical and an empirical motivation for the idea that this extension is important to better model the behavior of the management for reporting asset values. Both theoretical consideration and empirical analysis come to the conclusion that the correlation between the logarithm of the true asset value and the noise term can be different from zero. The empirical analysis showed that there is a negative correlation of -0.178 for a sample consisting of German companies. For a sub-sample that consists only of companies traded on *Neuer Markt* a correlation of -0.672 was estimated.

With the theoretical and empirical arguments at hand, I derived the conditional density of the true assets given the reduced information set consisting of the reported asset value and the information whether the company is in default. With the help of the conditional density, I calculated conditional probabilities of default, bond prices, and hypothetical credit spreads for corporate zero bonds. Furthermore, I showed that for usual parameters the correlation between the logarithm of the true asset value and the noise term is similar to the correlation between the true asset value and the noise term.

The analysis of the correlation effects demonstrates that introducing the correlation has two effects: a statistical and an information effect. The statistical effect is responsible for a new distribution of the reported asset value. The information effect arises from the anticipation of the correlation by the investor. In a numerical example it was shown that for small accounting noise the statistical effect dominates, while for medium and large accounting noise the information effect is dominant.

In the further analysis, I looked at the pricing errors in the credit spreads of corporate zero bonds that would arise if an investor ignored the correlation. For usual parametrizations, these pricing errors can have high magnitudes, especially for short times to maturity.

My analysis in this paper shows that a correlation between the logarithm of the true asset value and the noise term has to be incorporated into the basis model of Duffie and

Lando (2001). There is theoretical and empirical evidence for such a component, and the effects of the correlation are of significant magnitude.

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A Proof of Proposition 2

$Q(\tau > t|Y_t)$ is the integral of $b(z|y, z_0, t)$ with boundaries v_b and ∞ . From Proposition 1 it is known that $b(z|y, z_0, t)$ is given by

$$\begin{aligned} b(x|Y_t, z_0, t) &= \sqrt{\frac{\gamma_0}{\pi}} e^{-J(\tilde{y}, \tilde{x}, \tilde{z}_0)} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} \left(1 - e^{-\frac{2\tilde{z}_0\tilde{x}}{\sigma^2 t}}\right) \\ &= \underbrace{\sqrt{\frac{\gamma_0}{\pi}} e^{-J(\tilde{y}, \tilde{x}, \tilde{z}_0)} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)}}_{:=b_1} - \underbrace{\sqrt{\frac{\gamma_0}{\pi}} e^{-J(\tilde{y}, \tilde{x}, \tilde{z}_0)} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} e^{-\frac{2\tilde{z}_0\tilde{x}}{\sigma^2 t}}}_{:=b_2}, \end{aligned} \quad (23)$$

with

$$\begin{aligned} J(\tilde{y}, \tilde{x}, \tilde{z}_0) &= \frac{1}{2} \frac{1}{(1-\rho^2)} \left\{ \frac{(\tilde{y} - \tilde{x})^2}{a^2} + \frac{(\tilde{z}_0 + mt - \tilde{x})^2}{\sigma^2 t} - 2\rho \frac{(\tilde{y} - \tilde{x})(\tilde{z}_0 + mt - \tilde{x})}{a\sigma\sqrt{t}} \right\} \\ \gamma_0 &= \frac{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}{2a^2\sigma^2 t(1-\rho^2)} \\ \gamma_1 &= \frac{(\tilde{y} - (mt + \tilde{z}_0))^2}{a^2\sigma^2 t(1-\rho^2)}. \end{aligned}$$

As one can see in Equation (23), $b(z|y, z_0, t)$ consists of two parts that can be integrated separately. To integrate the first part with respect to x , we have to bring the expression into the form of the density of a normally distributed variable. Therefore, the exponent has to have the form

$$-\frac{(A_1\tilde{x} - Q_1)^2}{2D_1} = -\frac{1}{2D_1} (A_1^2\tilde{x}^2 - 2A_1\tilde{x}Q_1 + Q_1^2). \quad (24)$$

The values of A_1 , Q_1 , and D_1 are found by comparing the coefficients of $\frac{(A_1\tilde{x} - Q_1)^2}{2D_1}$ and $J(\tilde{y}, \tilde{x}, \tilde{z}_0)$. The values are

$$\begin{aligned} A_1 &= \sqrt{\sigma^2 t + a^2 - 2a\sigma\sqrt{t}\rho} \\ Q_1 &= \frac{\tilde{y}(\sigma^2 t - a\sigma\sqrt{t}\rho) + (\tilde{z}_0 + mt)(a^2 - a\sigma\sqrt{t}\rho)}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}} \\ D_1 &= a^2\sigma^2 t(1-\rho^2). \end{aligned}$$

With the parameters A_1 , Q_1 , and D_1 the exponent in Equation (23) equals the exponent (except for a correction term) in the density of a normally distributed variable with mean Q_1/A_1 and variance D_1/A_1^2 . The correction term C_1 can be written as

$$C_1 = \frac{((\tilde{y} - (\tilde{z}_0 + mt))^2}{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}.$$

To write b_1 as the density of a normally distributed variable, I correct the term in front of the exponent. Using all information so far, b_1 can be written as

$$b_1 = \underbrace{\frac{\sqrt{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} e^{-\frac{1}{2}C_1}}_{\text{constant}} \phi\left(\frac{\tilde{x} - \frac{Q_1}{A_1}}{\frac{D_1}{A_1^2}}\right), \quad (25)$$

where $\phi(\cdot)$ denotes the density of a standard normally distributed variable. Obviously, it is possible to rewrite b_1 as a constant times the density of a standard normal variable.

Rewriting the second part of Equation (23) is more difficult as it additionally involves x in $e^{\frac{-2\tilde{z}_0\tilde{x}}{\sigma^2 t}}$. Again, I rewrite b_2 as a normal density times a constant. Comparing coefficients in the exponential part of b_2 , I calculate the values for A_2 , Q_2 , and D_2 . This yields

$$\begin{aligned} A_2 &= \sqrt{\sigma^2 t + a^2 - 2a\sigma\sqrt{t}\rho} \\ Q_2 &= \frac{\tilde{y}(\sigma^2 t - a\sigma\sqrt{t}\rho) + \tilde{z}_0(a^2(1 - 2(1 - \rho^2)) - a\sigma\sqrt{t}\rho) + mt(a^2 - a\sigma\sqrt{t}\rho)}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}} \\ D_2 &= a^2\sigma^2 t(1 - \rho^2). \end{aligned}$$

The correction term C_2 of the exponent of b_2 is

$$C_2 = \frac{(\tilde{y} - (\tilde{z}_0 + mt))^2 + 4\tilde{y}\tilde{z}_0}{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho} + 4a^2 \frac{a^2\rho^2\tilde{z}_0^2 + a^2\tilde{z}_0mt - a\sigma\sqrt{t}\rho(\tilde{z}_0^2 + \tilde{z}_0mt + \tilde{y}\tilde{z}_0)}{(a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho)a^2\sigma^2 t}.$$

The term b_2 can now be rewritten as a constant times a normally distributed variable

$$b_2 = \underbrace{\frac{\sqrt{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}} e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} e^{-\frac{1}{2}C_2}}_{\text{constant}} \phi\left(\frac{\tilde{x} - \frac{Q_2}{A_2}}{\frac{D_2}{A_2^2}}\right), \quad (26)$$

where $\phi(\cdot)$ denotes the density of a standard normally distributed variable. Defining

$$\begin{aligned}
\gamma_2 &:= C_1 \\
\gamma_3 &:= C_2 \\
\gamma_6 &:= \frac{\sqrt{a^2 + \sigma^2 t + 2a\sigma\sqrt{t}\rho}}{\sqrt{a^2 + \sigma^2 t - 2a\sigma\sqrt{t}\rho}},
\end{aligned}$$

Equation (23) simplifies to

$$b(x|Y_t, z_0, t) = \gamma_6 e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} \left(e^{-\frac{1}{2}\gamma_2} \phi\left(\frac{\tilde{x} - \frac{Q_1}{A_1}}{\frac{D_1}{A_1^2}}\right) - e^{-\frac{1}{2}\gamma_3} \phi\left(\frac{\tilde{x} - \frac{Q_2}{A_2}}{\frac{D_2}{A_2^2}}\right) \right). \quad (27)$$

Rewriting the integral of $\phi(\cdot)$ as $\Phi(\cdot)$ and inserting the integration bounds yields

$$\int_{v_b}^{\infty} b(z|Y_t, z_0, t) dz = \gamma_6 e^{\frac{1}{4}\left(\frac{\gamma_1}{\gamma_0}\right)} \left(e^{-\frac{1}{2}\gamma_2} \Phi\left(\frac{Q_1 A_1}{D_1}\right) - e^{-\frac{1}{2}\gamma_3} \Phi\left(\frac{Q_2 A_2}{D_2}\right) \right). \quad (28)$$

Defining

$$\gamma_4 := \frac{Q_1 A_1}{D_1}$$

and

$$\gamma_5 := \frac{Q_2 A_2}{D_2}$$

gives the required result.

B Tables and Figures

Table 1: The base case parameters were taken from Panel A of Herkommer (2006). See Herkommer (2006) for a further discussion of the particular parameter choices.

Parameter	$T - t$	m	σ	r	V_b	V_0	F	ω
	10	0.07	0.15	0.04	60.0	86.3	100	0.5

Figure 1: In the two figures credit spreads for the model of Duffie and Lando (2001) are plotted for different levels of accounting noise a . In the first figure the reported asset value is set to 90. In the second figure the reported asset value is set to 72. For the other model parameters, the base case values are applied.

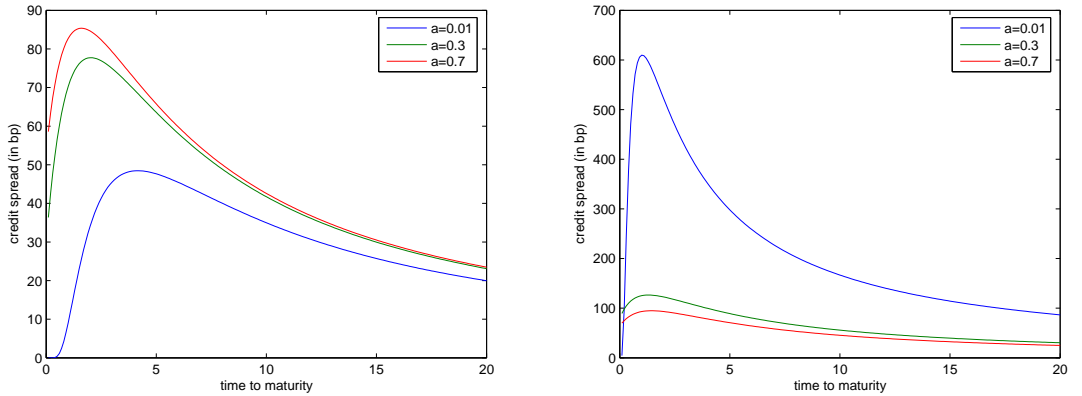


Figure 2: In the figures the dependence between the logarithm of the true asset value and the noise term are plotted for different sub-samples. In the left top figure the dependence structure for the whole sample of 3,035 companies is plotted. In the right top and in the middle left figure the dependences for sub-samples of *Neuer Markt* companies and other companies are plotted. In the middle right, lower left, and lower right figure the dependence structure for sub-samples of companies are plotted that report according to IAS, US GAAP, and German GAAP respectively.

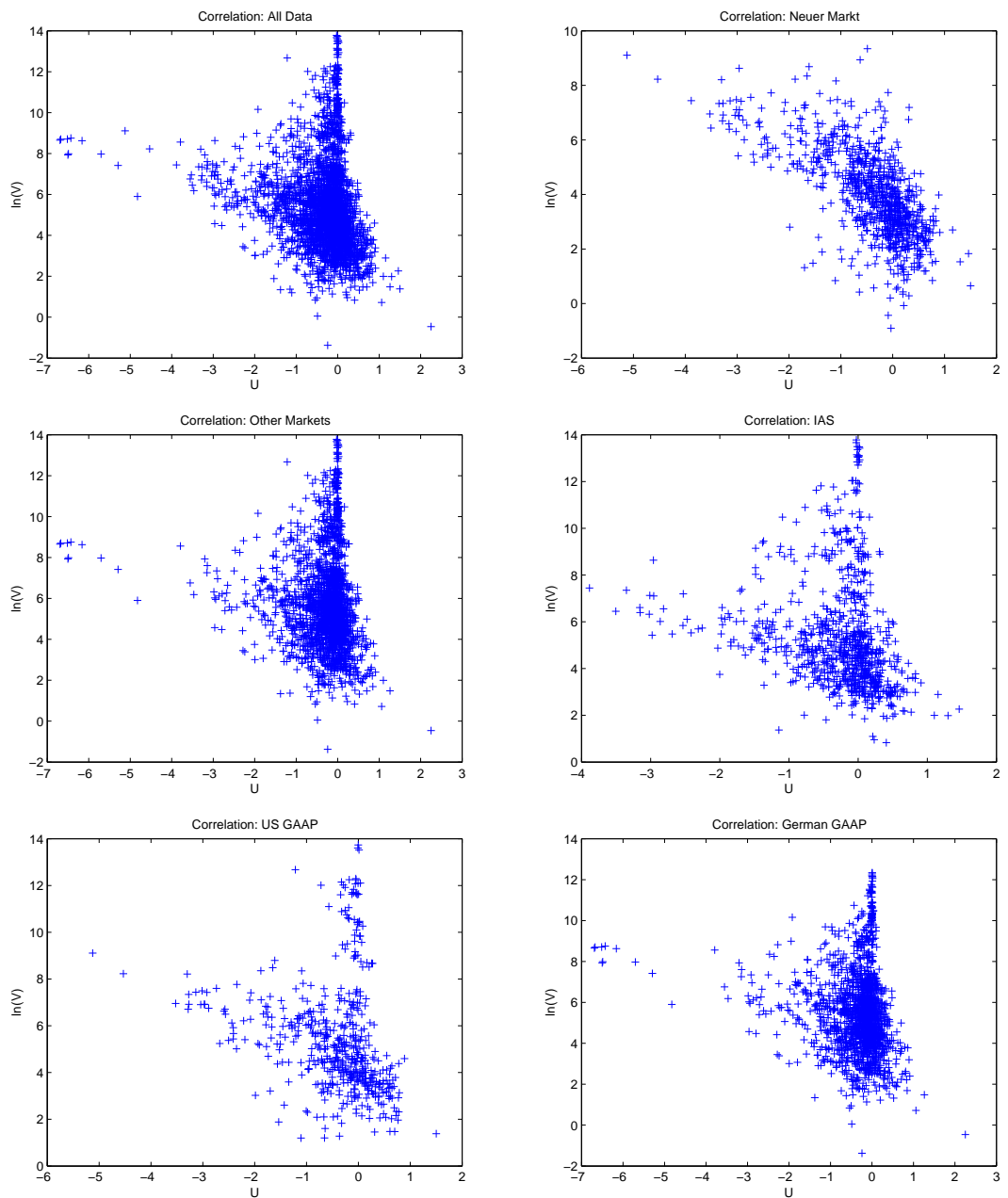


Figure 3: In this figure the correlation between the logarithm of the true asset value and the noise term is compared to the correlation of the true asset value and the noise term for three different levels of volatility.

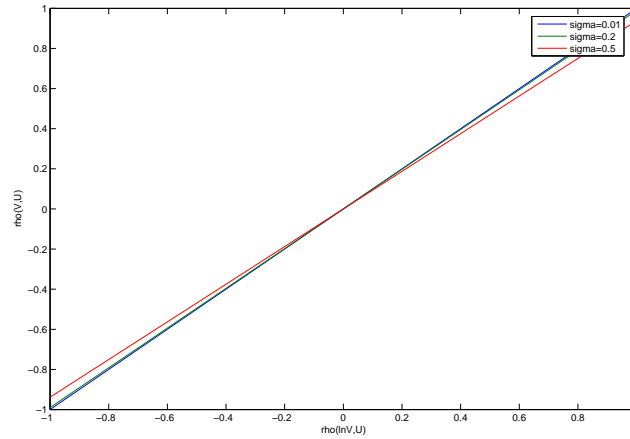


Figure 4: In this figure the effect of the correlation and the accounting noise on credit spreads is plotted for a reported asset value of 120. For small accounting noise a , credit spreads decrease as the correlation ρ increases. For medium and large accounting noise a , credit spreads increase as the correlation ρ increases.

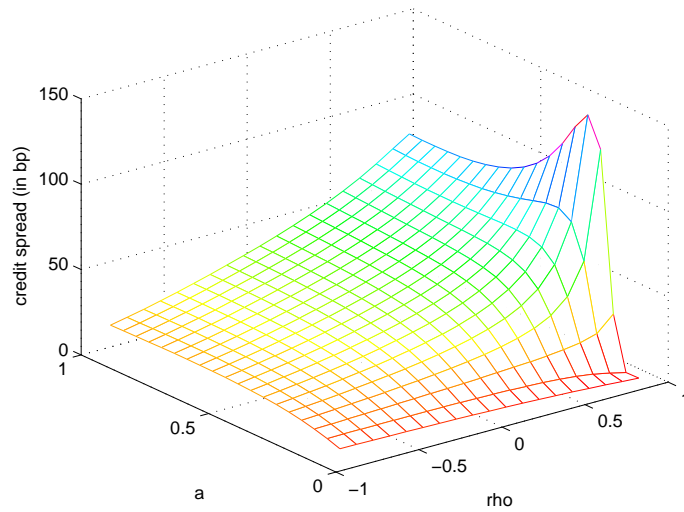


Figure 5: In this figure the effect of the correlation and the accounting noise on credit spreads is plotted for a reported asset value of 93.6. For small accounting noise a , credit spreads decrease as the correlation ρ increases. For medium and large accounting noise a , credit spreads increase as the correlation ρ increases.

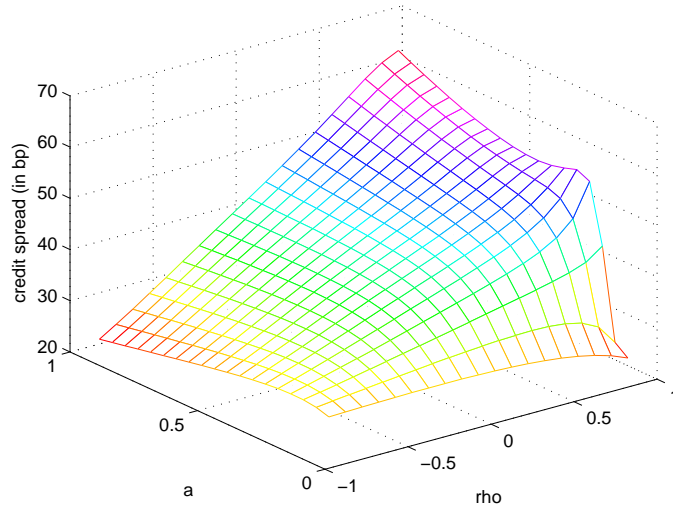


Figure 6: In this figure the effect of the correlation and the accounting noise on credit spreads is plotted for a reported asset value of 65. For small accounting noise a , credit spreads increase as the correlation ρ increases. For medium and large accounting noise a , credit spreads decrease as the correlation ρ increases.

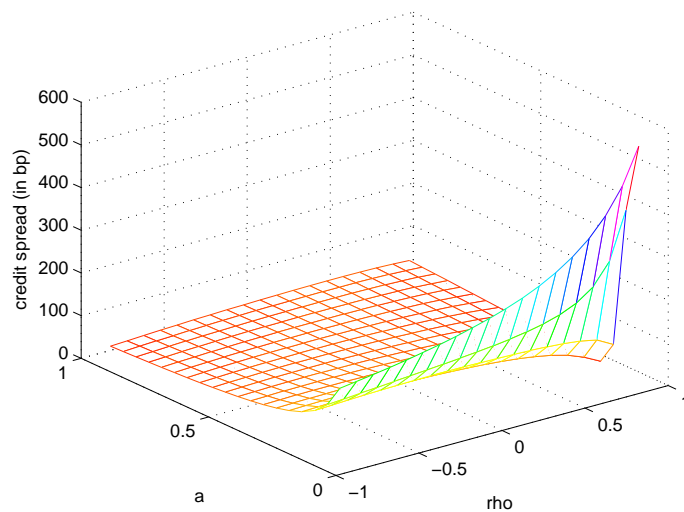


Figure 7: In the first row, the credit spreads and the pricing errors are plotted for the case that the reported asset value is 120. In the second row, the credit spreads and the pricing errors are plotted for the case that the reported asset value is 93.6. In the third row, the credit spreads and the pricing errors are plotted for the case that the reported asset value is 65. In each of these cases I examine three correlation scenarios of 0, -0.178 , and -0.672 .

