A STATE-OF-THE-ART REVIEW OF TRANSFORMER PROTECTION ALGORITHMS

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ABSTRACT

This paper presents a comparative study of algorithms for digital differential protection of power transformers. The mathematical basis for each algorithm is briefly described. The algorithms are compared as to their speed of response, computational burden and the capability to distinguish between an inrush and transformer internal fault. This theoretical comparison of algorithms for transformer digital protection is presented in a comprehensive form for the first time.

INTRODUCTION

For the last 15 years, there has been considerable interest in the area of digital protection of power apparatus [1]. In the early 1970s dedicated digital relays using minicomputers were proposed. Some utilities also tested minicomputer based experimental on-line systems for digital protection of transmission lines in their substations [2,3]. The main features which have encouraged many researchers to investigate the feasibility of designing digital relays for power system protection are its economy, reliability, flexibility, improved performance over conventional relays and the possibility of integrating a digital relay into the hierarchial computer system within the substation. The introduction of microprocessors has brought about novel and low-cost possibilities for the development of protection devices for power systems and power apparatus. The capability of the currently available microprocessors is such that all the digital relaying functions performed by the minicomputers of the 1970s can now be done by microprocessor systems. As a result, many investigators published results of work on specific hardware and software techniques for microprocessor based transmission line relays. A few prototype distance relays using multiple microprocessors have also been tested on-line [4,5].

As the utilities gain experience with the use of these prototype digital relays, one can foresee that low cost, dedicated microprocessor based relays will appear in commercial service during the next few years. Considerable attention has not been given to the on-line implementation of digital power transformer protection. Only during the last few years, researchers have been investigating microprocessor based three phase transformer relays [6,7]. The technical literature available on this subject is mainly on the algorithms for power transformer protection [10-18].

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Various claims have been made on the speed, accuracy, computational burden, etc. of these algorithms. It seems quite difficult to grasp the real significance of these algorithms since they are evaluated using different models. The purpose of this paper is to provide a state of the art review of most of the algorithms for digital protection of power transformer. The algorithms are evaluated on a common basis. The paper also identifies the algorithms which may be implemented in a practicel design.

PRINCIPLE OF TRANSFORMER PROTECTION

The differential relaying principle [8] is commonly used for the protection of power transformers. This is based on a comparison of the primary and secondary currents. These currents are in a known relationship under healthy conditions. When the currents deviate from the pre-defined relationship an internal fault is assumed and the transformer is switched off. This method is quite satisfactory in most respects but is subject to false tripping on the transient magnetizing inrush current which flows when the transformer is energized. This current flows in one winding only of the transformer (the winding being energized) and therefore appears to the protection as an internal fault condition. The most common technique used for preventing false trips during energization is the use of a harmonic restraint relay [9]. The inrush current differs from an internal fault current in so far as its waveform comprises a high percentage of harmonics. Of these, the second harmonic has particular prominence under all switching conditions. This fact is used in the design of a harmonic restraint relay. If the second harmonic content of the differential current exceeds a pre-defined percentage of the fundamental, inrush is assumed and the protection is prevented from tripping.

The published literature on the digital protection of power transformers has received attention as far as the development of algorithms for differential protection is concerned. These algorithms sample the differential current at regular time intervals and determine its fundamental and harmonic content. The decision as to trip or restrain is then based on the relative levels of the fundamental and second harmonic content. Sykes and Morrison [10] suggested the use of recursive band pass filters to calculate the fundamental and second harmonic content. Malik, Dash and Hope [11] proposed the application of a cross correlation approach based on Fourier techniques. Schweitzer et al [12] proposed finite impulse response (FIR) filters to calculate the magnitude of the fundamental and second harmonic components. Degens [13] and Rahman et al. [14] presented algorithms based on the least-squares curve fitting technique to find the second harmonic and fundamental components of the differential current. Rahman and Dash [15] have presented an algorithm using the rectangular transform technique, which involves a few number of multiplications. In a recent paper [16] Fakruddin et al. have investigated the application of Haar functions for differential protection.

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Application of Walsh functions has also been proposed by Rahman et al. [17,18] for digital protection of transformers. The following section summarizes the mathematical formulation of the commonly used algorithms for transformer differential protection.

ALGORITHMS FOR TRANSFORMER DIFFERENTIAL PROTECTION

Fourier Analysis Approach [11]

Any waveform g(t) having a finite energy in the interval (0,T) can be expressed in the interval as a Fourier Series:

$$g(t) = F_0 + \sqrt{2}\, F_1 \sin(\frac{2\pi t}{T}) + \sqrt{2}\, F_2 \cos(\frac{2\pi t}{T}) + \sqrt{2}\, F_3 \sin(\frac{4\pi t}{T}) + \sqrt{2}\, F_4 \cos(\frac{4\pi t}{T}) + \ldots \quad (1)$$

with coefficients

$$F_0 = \frac{1}{T} \int_0^T g(t)\, dt$$

$$F_1 = \frac{\sqrt{2}}{T} \int_0^T g(t) \sin(\frac{2\pi t}{T})\, dt \quad (2)$$

$$F_2 = \frac{\sqrt{2}}{T} \int_0^T g(t) \cos(\frac{2\pi t}{T})\, dt$$

$$\vdots$$

Using this approach, the differential current is analyzed in terms of its Fourier series and the peak of the fundamental and second harmonic contents are given by:

$$\text{AMP1} = \sqrt{2}\, (F_1^2 + F_2^2)^{1/2}$$
$$\text{AMP2} = \sqrt{2}\, (F_3^2 + F_4^2)^{1/2} \quad (3)$$

$F_1 \ldots F_4$ are obtained by correlation of sampled differential current with stored samples of reference sine and cosine waves.

Rectangular Transform Approach [15]

From the samples of the differential current $g(t_i)$, Fourier sine and cosine coefficients can also be defined as:

$$S_k = \sum_{j=0}^{n-1} g(t_j) \sin(\frac{2\pi jk}{n}) \quad (4)$$

$$C_k = \sum_{j=0}^{n-1} g(t_j) \cos(\frac{2\pi jk}{n}) \quad (5)$$

If the sine and cosine terms in equation (4) and (5) are replaced with the equivalent 'rectangular' functions [5]

$$\sin \gamma t(x) = \text{sgn}(\sin x) \quad (6)$$

$$\cos \gamma t(x) = \text{sgn}(\cos x) \quad (7)$$

where

$$\text{sgn}(y) = y/|y| \quad \text{for } y \neq 0$$
$$= 0 \quad \text{for } y = 0$$

then corresponding 'rectangular transform' coefficients will be denoted as S_{KR} and C_{KR}.

From the samples $g(t_i)$, rectangular coefficients can be obtained as:

$$[S_{KR}] = [A]\, [S_K] \quad (8)$$

$$[C_{KR}] = [B]\, [C_K] \quad (9)$$

In equation (8), $[S_K]$ is the vector of Fourier sine coefficients and $[S_{KR}]$ is the vector of corresponding rectangular coefficients. The Fourier coefficients can be calculated from the rectangular coefficients as

$$[S_K] = [A^{-1}]\, [S_{KR}] \quad (10)$$

$$[C_K] = [B^{-1}]\, [C_{KR}] \quad (11)$$

For example, the matrix $[A^{-1}]$ takes the form:

$$[A^{-1}] = \begin{bmatrix} 1 & 0 & -1/3 & 0 & -1/5 & 0 & \cdots \\ & 1 & 0 & 0 & 0 & -1/3 & \cdots \\ & & 1 & 0 & 0 & 0 & \cdots \\ & & & 1 & 0 & 0 & \cdots \\ & & & & 1 & 0 & \cdots \\ & & & & & & \cdot \\ & & & & & & \cdot \end{bmatrix}$$

Using this approach for transformer differential protection, $F_1 \ldots F_4$ can be obtained as:

$$F_1 = S_{1R} - 1/3\, S_{3R} - 1/5\, S_{5R} \quad (12)$$

$$F_2 = C_{1R} + 1/3\, C_{3R} - 1/5\, C_{5R} \quad (13)$$

$$F_3 = S_{2R} \quad (14)$$

$$F_4 = C_{2R} \quad (15)$$

S_{1R}, C_{1R}, ... are the rectangular coefficients obtained from sampled differential current $g(t_i)$, using equation (8) and (9). The fundamental and second harmonic content of the differential current can now be determined using equation (3).

Algorithm based on Walsh Functions [17,18]

Walsh functions form a complete set of orthogonal functions. These functions which are denoted by wal(k,t) bear a number of resemblances to sine and cosine functions appearing as the squared up versions of them. They take only the values ±1 and change sign only when t is a multiple of a power of 1/2. In terms of these functions, the Walsh expansion of g(t) in the interval (0,T) is defined as

$$g(t) = \sum_{k=0}^{\infty} W_k \,\text{wal}(k, \frac{t}{T}) \quad (16)$$

where

$$W_k = \frac{1}{T} \int_0^T g(t)\, \text{wal}(k, \frac{t}{T})\, dt \quad (17)$$

The Walsh coefficients (W_k) are obtained by correlating the samples of differential current $g(t_i)$ with reference Walsh functions. The Walsh method is convenient to implement only for a number of samples which is an integral power of two. The relationship between Fourier and Walsh coefficients can be expressed in matrix form as

$$W = AF \tag{18}$$

where any element A_{kk} of matrix A is the result of substituting the kth sinusoid that appears in equation (1) for g(t) in (16). The Fourier coefficients F_1, F_2, F_3 and F_4 can be determined from the Walsh coefficients as:

$$F_1 = 0.9\ W_1 - 0.373\ W_5 - .074\ W_9 \tag{19}$$

$$F_2 = 0.9\ W_2 + 0.373\ W_6 - .074\ W_{10} \tag{20}$$

$$F_3 = 0.9\ W_3 - 0.373\ W_{11} \tag{21}$$

$$F_4 = 0.9\ W_4 + 0.373\ W_{12} \tag{22}$$

The Walsh coefficients $W_1 \ldots W_{12}$ are obtained by addition and subtraction of the signal samples only in accordance with the reference square waves. Once F_1, F_2, F_3, F_4 are available, the fundamental and second harmonic content of the differential current can be easily obtained.

Haar Functions Approach [16]

The Haar function set forms a complete set of orthogonal rectangular functions. Haar functions have three possible states 0, +A and -A where ±A is a function of $\sqrt{2}$. They may be expressed as HAR(n,t).

For a time interval $0 < t < 1$, the functions are given as

$$\mathrm{HAR}(0,t) = 1 \qquad \text{for } 0 < t < 1$$

$$\mathrm{HAR}(1,t) = \begin{cases} 1 & 0 < t < 1/2 \\ -1 & \text{for } 1/2 < t < 1 \end{cases}$$

$$\mathrm{HAR}(2^p+n,t) = \begin{cases} \sqrt{2}^p & \text{for } n/2^p < t < (n+1/2)/2^p \\ -\sqrt{2}^p & (n+1/2)/2^p < t < (n+1)/2^p \\ 0 & \text{elsewhere} \end{cases}$$

$$p = 1, 2, \ldots \qquad n = 0, 1, 2, \ldots (2^p-1)$$

In terms of these functions, the Haar expansion of g(t) in the interval (0,T) is defined as

$$g(t) = \sum_{k=0}^{\infty} C_K\ \mathrm{HAR}(k,t) \tag{23}$$

where

$$C_K = \int_{t=0}^{1} g(t).\ \mathrm{HAR}(k,t)\ dt \tag{24}$$

The Haar coefficients (C_K) are obtained by correlating the samples of differential current $g(t_i)$ with reference Haar functions as given by equation (24). The Fourier coefficients $F_1 \ldots F_4$ are then obtained as

$$F_1 = .9\ C_1 + 0.1865(-C_4 + C_5 + C_6 - C_7) + 0.089(-C_8 + C_{11} + C_{12} - C_{15}) + 0.037(-C_9 + C_{10} + C_{13} - C_{14}) \tag{25}$$

$$F_2 = 0.6366(C_2 - C_3) + 0.1865(C_4 + C_5 - C_6 - C_7) + 0.037(C_8 + C_{11} - C_{12} - C_{15}) + 0.089(C_9 + C_{10} - C_{13} - C_{14}) \tag{26}$$

$$F_3 = .6366(C_2 + C_3) + 0.1318(C_9 + C_{10} + C_{13} + C_{14} - C_8 - C_{11} - C_{12} - C_{15}) \tag{27}$$

$$F_4 = .6366(C_4 - C_5 + C_6 - C_7) + .1318(C_8 + C_9 + C_{12} + C_{13} - C_{10} - C_{11} - C_{14} - C_{15}) \tag{28}$$

As in the previous algorithms, the fundamental and second harmonic content can then be determined.

Finite Impulse Response Approach [12]

In this approach four finite impulse response (FIR) filters are used; two each for the fundamental and second-harmonic. The filters cover one cycle of time and have values of either plus or minus one at any instant during that period. The impulse responses are restricted to plus or minus one so that finding the responses of the filters to arbitrary inputs is reduced to adding together samples of the input, with appropriate sign changes as determined by the impulse responses. Using *time-discrete differetial* current samples, i_m, sampled N times per cycle, the filter outputs are calculated as

$$F_1(K) = \sum_{m=k-N+1}^{k-N/2} [i_m - i_{m+N/2}] \tag{29}$$

$$F_2(k) = \sum_{m=k-N+1}^{k-3N/4} [i_m - (i_{m+N/4} + i_{m+N/2}) + i_{m+3N/4}] \tag{30}$$

$$F_3(k) = \sum_{m=k-N+1}^{k=3N/4} [i_m - i_{m+N/4} + i_{m+N/2} - i_{m+3N/4}] \tag{31}$$

$$F_4(k) = \sum_{m=k-N+1}^{k-7N/8} [i_m - (i_{m+N/8} + i_{m+N/4}) + i_{m+3N/8} + i_{m+N/2} - (i_{m+5N/8} + i_{m+3N/4}) + i_{m+7N/8}] \tag{32}$$

where k stands for the kth sample period. The fundamental and second harmonic contents are then given by equation (3).

Least-Squares Curve Fitting Algorithm [13]

In this approach, the inrush current is assumed to contain the fundamental frequency component, a decaying DC component and upto 5th harmonic. In a certain time the inrush current can be written as

$$i_1(t) = p_o \exp(-\lambda t) + \sum_{k=1}^{5} p_k \sin(k\ W_o t + \Theta_k) \tag{33}$$

If the time window is small compared with the time constant $1/\lambda$ of the decaying DC, the DC component can be approximated by

$$p_o \exp(-\lambda t) \cong p_o - p_o \lambda t \tag{34}$$

Equation (33) simplifies to

$$i_1(t) = p_o - p_o\lambda t + \sum_{k=1}^{5} p_k \cos\Theta_k \sin kw_o t + \sum_{k=1}^{5} p_k \sin\Theta_k \cos kw_o t \qquad (35)$$

If N samples of the differential current are considered at time $t_1, t_2, \ldots, t_N$, the sampling process results in the following set of equations:

$$\begin{bmatrix} 1 & t_1 & \sin w_o t_1 & \cos w_o t_1 \cdot \sin 5w_o t_1 & \cos 5w_o t_1 \\ 1 & t_2 & \sin w_o t_2 & & \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 1 & t_N & & & \cos 5w_o t_N \end{bmatrix} \begin{bmatrix} p_o \\ -p_o\lambda \\ p_1\cos\Theta_1 \\ p_1\sin\Theta_1 \\ \cdot \\ \cdot \\ p_5\sin\Theta_5 \end{bmatrix} = \begin{bmatrix} i(t_1) \\ i(t_2) \\ \cdot \\ \cdot \\ \cdot \\ i(t_N) \end{bmatrix} \qquad (36)$$

As the vector of the unknown contain 12 elements, it is necessary that N > 12 to solve the unknown. If equation (36) is written as

$$[A] \cdot X = I \qquad (37)$$

then the least-squares solution of equation (37) is given by

$$X = [(A^t \cdot A)^{-1} \cdot A^t] \cdot I = B \cdot I \qquad (38)$$

From equation (38) it follows that matrix B can be calculated by

$$B = [A^t \cdot A]^{-1} \cdot A^t \qquad (39)$$

The elements that occur in matrix B can be calculated off-line from equation (39). Matrix B that results contains known elements, which can be used to calculate the vector of unknown X from the sampled differential current. To determine the amount of fundamental and second harmonic present in the differential current, only the unknown $p_1 \cos\Theta_1$, $p_1 \sin\Theta_1$, $p_2 \cos\Theta_2$ and $p_2 \sin\Theta_2$ in equation (36) are of importance. This can be found by

$$\begin{aligned} p_1 \cos\Theta_1(t_N) &= \sum_{n=1}^{N} B(3,n)\, i(t_n) \\ p_1 \sin\Theta_1(t_N) &= \sum_{n=1}^{N} B(4,n)\, i(t_n) \\ p_2 \cos\Theta_2(t_N) &= \sum_{n=1}^{N} B(5,n)\, i(t_n) \\ p_2 \sin\Theta_2(t_N) &= \sum_{n=1}^{N} B(6,n)\, i(t_n) \end{aligned} \qquad (40)$$

The peak of the fundamental and second harmonic can then be calculated as

$$p_1 = (p_1^2\cos^2\Theta_1 + p_1^2\sin^2\Theta_1)^{1/2} \qquad (41)$$

$$p_2 = (p_2^2\cos^2\Theta_2 + p_2^2\sin^2\Theta_2)^{1/2} \qquad (42)$$

It has been found that a sampling frequency of 12 samples per cycle, data window (N) of 13 samples and time reference $t_1 = 0$ needs the lowest number of multiplications to yield the four unknowns of equation (40). This method is basically a process of digital filtering. Every time a new sample is taken, the old ones are shifted and digital filtering according to equation (40) is carried out again on the new set of samples.

FREQUENCY RESPONSE OF ALGORITHMS

All the above mentioned algorithms may be considered as implementation of non recursive digital filtering techniques. The objective of each algorithm is to determine the peak of the fundamental and harmonic content of the differential current as each new sample is taken. The digital filter outputs depend on the present and past input signal samples only. Only the filter weighing factors are unique to each algorithm. The frequency response plot of the Fourier Algorithm, FIR algorithm and Curve fitting algorithm are shown in Figures 1-3. Frequency

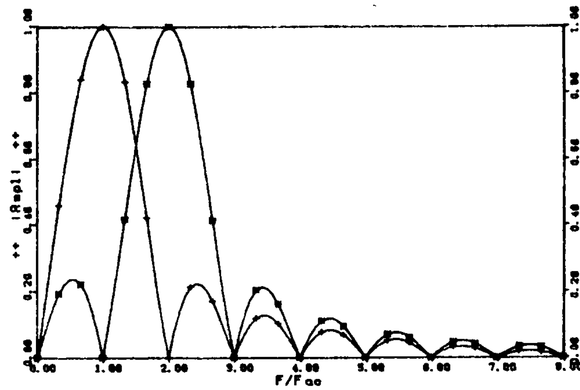

Figure 1 Frequency Response of Fourier Algorithm

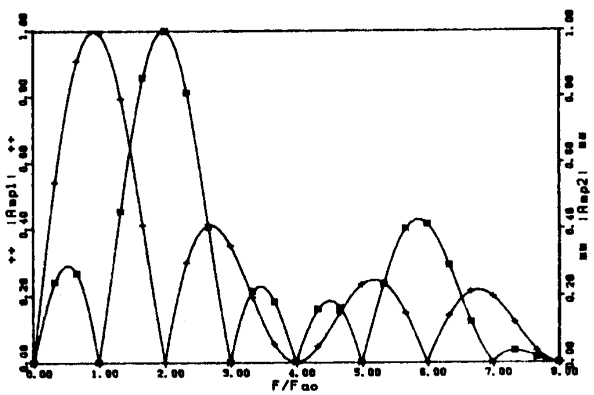

Figure 2 Frequency Response of Finite Impulse Response Algorithm

response is a measure of the filtering characteristic of the algorithm. By transforming the algorithm into the complex-z domain and substituting frequency for the complex variable z, the frequency response plot is obtained. This plot is symmetric about the Nyquist frequency (one-half the sampling frequency) and repeats after each integer multiple of the sampling frequency. The frequency response plots for Rectangular transform, Walsh functions and Haar functions are very similar to that of the Fourier algorithm since all these algorithms basically obtain F_1, F_2, F_3 and F_4 of equation (1) by correlating the

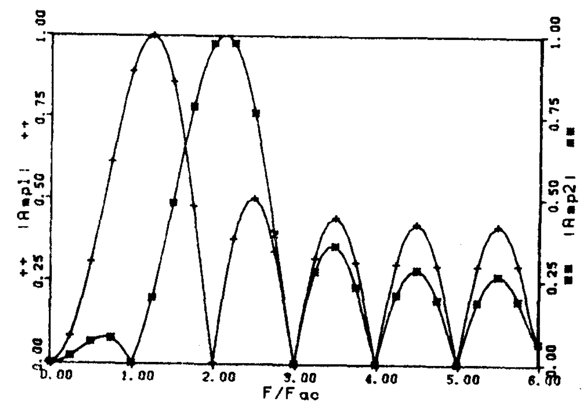

Figure 3 Frequency Response of Curve Fitting Algorithm

signal samples with reference waveforms. The frequency response plots show that all these algorithms are able to effectively extract components of the fundamental and second harmonic frequencies. Each algorithm may be considered as four digital filtering computations, two each for fundamental ad second harmonic. In the frequency response plots shown in Figures 1-3, the frequency response of the two filters for fundamental are combined into a single response (AMP1) and similarly for the second harmonic filter (AMP2). The responses are plotted as a function of normalized frequency ratio of F/F_{ao}, where F_{ao} is the base frequency. The fundamental amplitlude AMP1 shown in the + marked curves has peak at fundamental frequency. Similarly, AMP2 shown in the ◘ marked curves, has its peak at second harmonic frequency of 2.0 per unit. The AMP 2 response of Finite Impulse algorithm has relatively higher minor peak at 6th harmonic frequency as shown in Fig. 2.

TRANSFORMER MODEL FOR ALGORITHM EVALUATION

In the work on digital differential protection of transformers proposed in the past, a single phase transformer and its model were used for testing the prototype relays. In this paper, a digital computer model of a three phase transformer is considered for evaluating the algorithms. The amount of inrush depends on the polarity of the voltage waveform, switching angle and residual flux associated with the respective cores. Earlier studies [19] on inrush in 3-phase transformers show that there may be an inrush current into more than one phase, and these separate inrush currents may be affected by the electric connections and/or magnetic coupling between phases. When a star-delta transformer is energized from the star side, the inrush current in two phases aid the third phase and as a result, the current in one phase is oscillatory. This phenomena is called 'helping effect'. In differential relaying it is almost universal practice to connect the current transformers in delta on the star-side of a star-delta bank. This connection eliminates zero-sequence current from the relay, and yields the correct phase angles to go with star-connected current transformers on the delta side of the bank. Thus the current flowing to any one relay is the difference between the magnetizing currents in two phases. The amount of helping effect is of no consequence, since it has disappeared in the delta relay currents. The current transformers on the delta side of a star-delta-connected bank are usually connected in star when they are used to energize differential relaying. Thus the relays receive the phase currents. The relay current is the difference between the total magnetising current in two phases. Hence, the inrush phenomena, from a relay point of view is the same with star-connected current transformers measuring inrush currents into a delta winding as it is with delta-connected current transformers measuring inrush currents into a star-connected winding.

In this simulation, the values of the inrush currents in individual phases are taken as independent inrushes [20]. The inrush current in each phase is obtained using Specht's [21] formula. As no mutual effect is considered between the limbs in the simulation, it is assumed that three single phase transformers are electrically connected, but magnetically isolated to achieve three-phase representation.

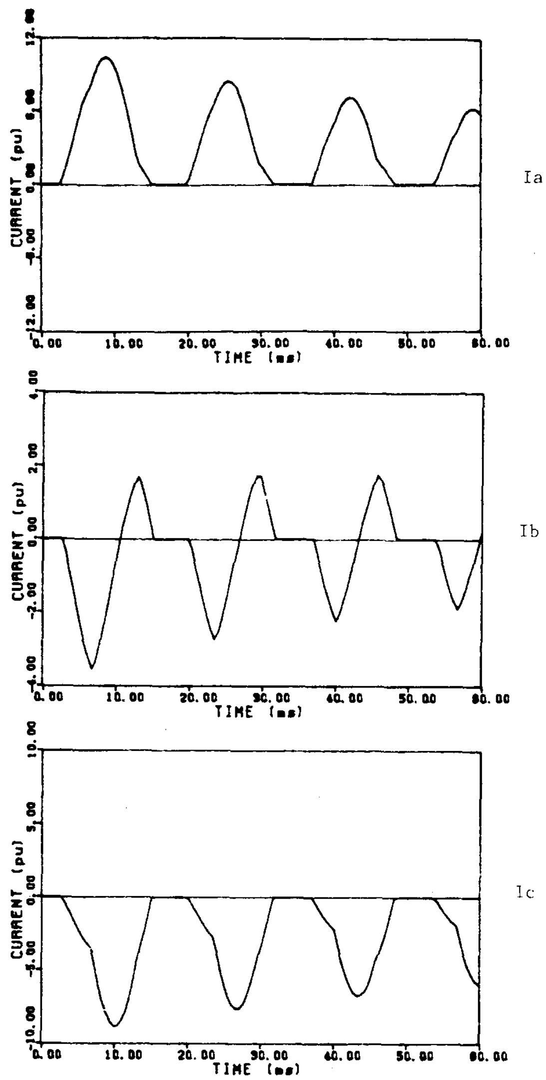

Figure 4 Inrush Current on Star Side of a Star-Delta Transformer

Figure 4 shows inrush current of a star-delta transformer energized from the star side. It is assumed that the switching is done when the phase A voltage angle λ is zero. Phase A is assumed to have a remnant flux equal to 90% of the peak steady-state flux and phase C is assumed to have a remnant flux equal to 50% of the peak steady-state flux. The core

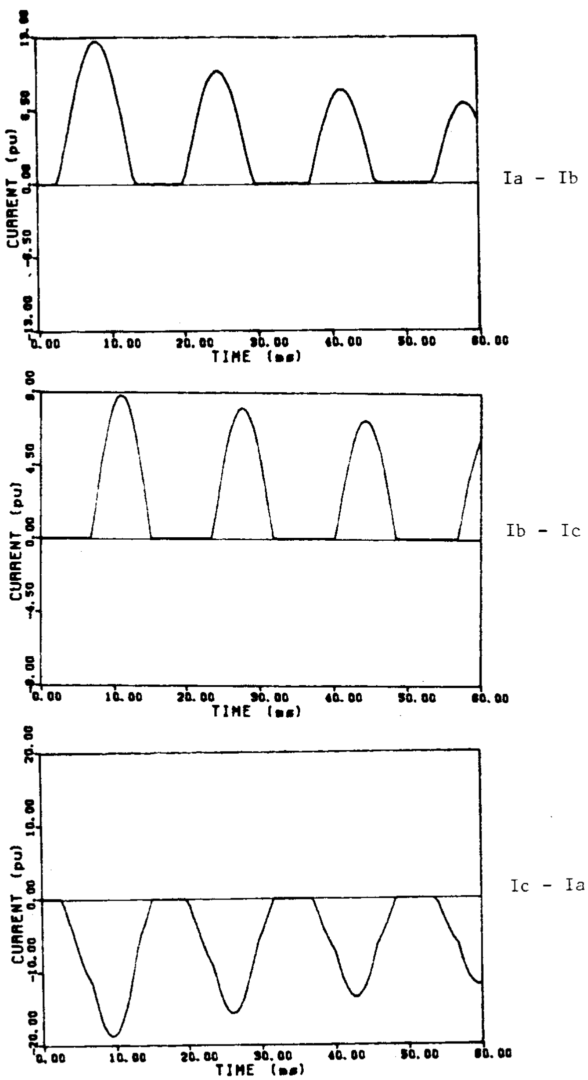

Figure 5 Relay Currents (Delta) Corresponding to Figure 4

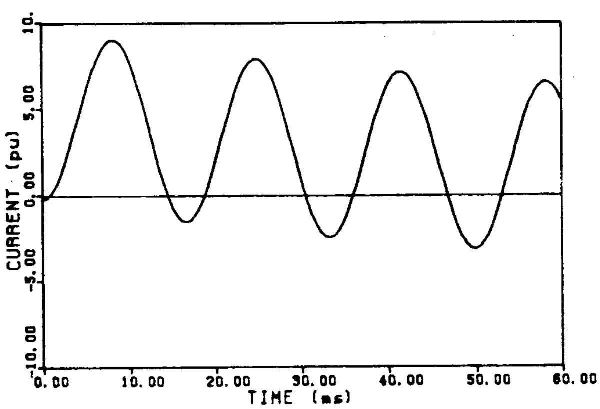

Figure 6 Transformer Internal Fault Current (Phase A)

is assumed to saturate at a flux level equal to 140% of the peak steady-state flux. From Figure 4 it can be seen that the phase B current is oscillatory. Figure 5 shows the corresponding relay currents which are obtained by connecting the current transformers in delta on the star side of the transformer.

Figure 6 shows phase A fault current for a 3-phase internal fault. Before the fault occurs power is delivered by the source to a load through the transformer. The fault-current is seen to have a very high offset.

SIMULATION RESULTS

A three phase transformer is simulated based on the previously discussed model [20] using a VAX-11/785 mainframe computer. The simulations provide samples of currents in each phase when the transformer is energized or when an internal fault occurs. For simulation purposes, three single phase transformers are electrically connected, but magnetically isolated to form the 3-phase representation. Data from the simulations are used as input to the different algorithms to identify the response of the algorithms. A sampling frequency of 960 Hz has been considered for all the algorithms except curve fitting algorithm. For the curve fitting algorithm a sampling frequency of 720 Hz has been considered in order to simplify the computations.

It is reported in literature [22] that a secure restraint function for a three-phase transformer can be obtained if a single harmonic restraint signal is derived by combining the harmonics of the three phases. In the simulation results shown here, the restraining signal is based on the combined second harmonic content of the relay currents in the three phases and the operating signal is based on the combined fundamental component.

The performance of all the algorithms for inrush is shown in Figures 7(a) to (f). The inrush currents in all three phases and the corresponding relay currents are shown earlier in Figures 4 and 5 respectively. As seen from Figure 7, the restraining signal is well above the operating signal and hence there is no possibility of relay maloperation. The performance of all the algorithms for a 3-phase internal fault is shown in Figures 8(a) to 8(f). Phase A current for this simulation is shown earlier in Figure 6. The restraining signal is greater than the operating signal for the signal samples immediately following the fault. This is due to the fact that the algorithm initializes the data window with signal samples of 0 magnitude. As the data window is filled with actual signal samples of the differential current, the operating signal becomes greater than the restraining signal and a secure trip decision is obtained.

COMPARISON OF ALGORITHMS

Before designing a microprocessor based differential relay for transformer protection, one should consider the computational requirements for the different algorithms and evaluate the capability of the microprocessors available in the market. In a transformer digital relay the transformer current in each phase is sampled at regular intervals and stored in the processor memory through sample-hold circuits and analog to digital converter. During each sampling interval the fundamental and second harmonic content of the differential currents are evaluated using the particular algorithm. The relay logic then determines the operating and restraining signals. If a fault is

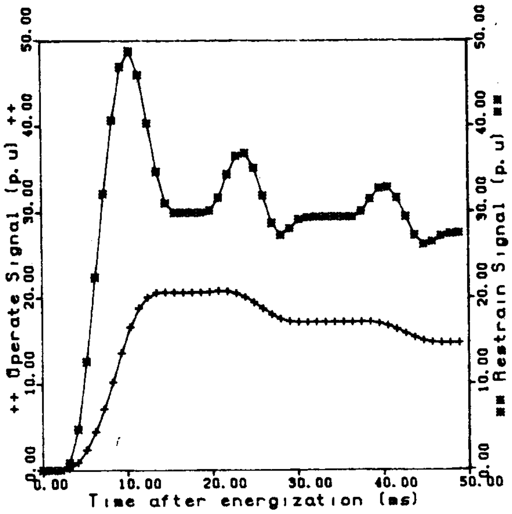

Figure 7.a Fourier Algorithm Performance for Inrush

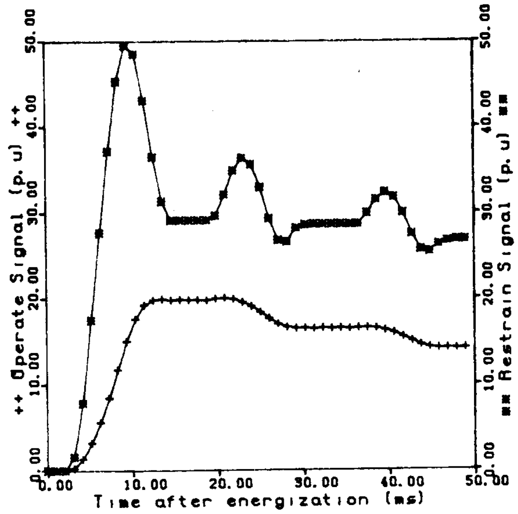

Figure 7.d Haar Functions Algorithm Performance for Inrush

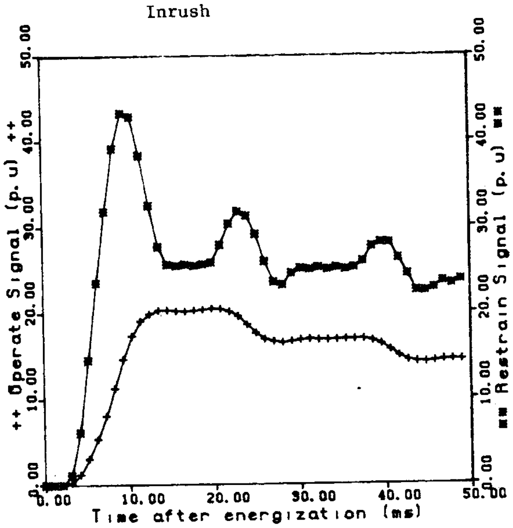

Figure 7.b Rectangular Transform Algorithm Performance for Inrush

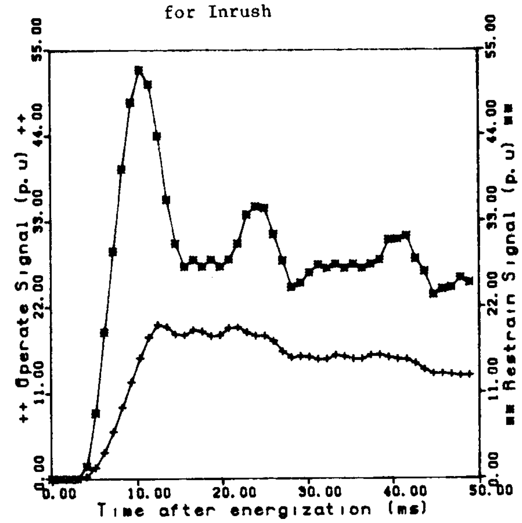

Figure 7.e Finite Impulse Response Algorithm Performance for Inrush

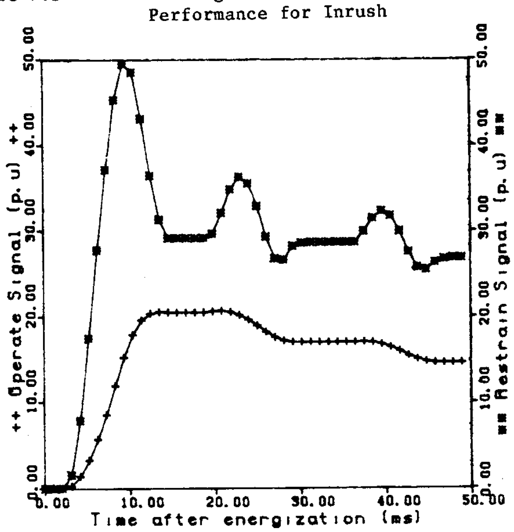

Figure 7.c Walsh Functions Algorithm Performance for Inrush

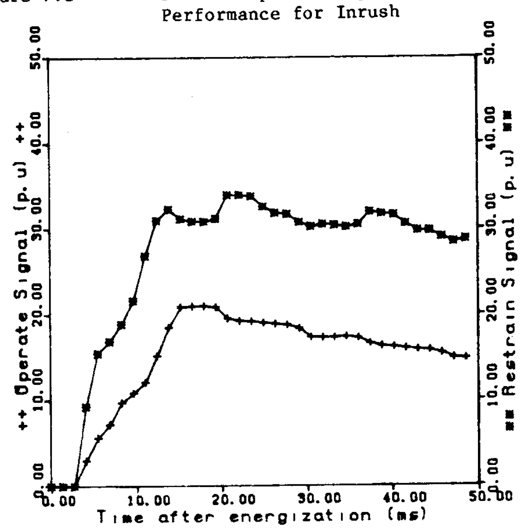

Figure 7.f Curve Fitting Algorithm Performance for Inrush

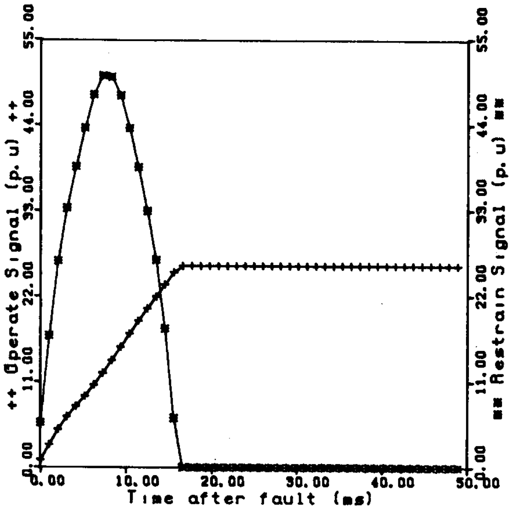

Figure 8.a Fourier Algorithm Performance for Fault

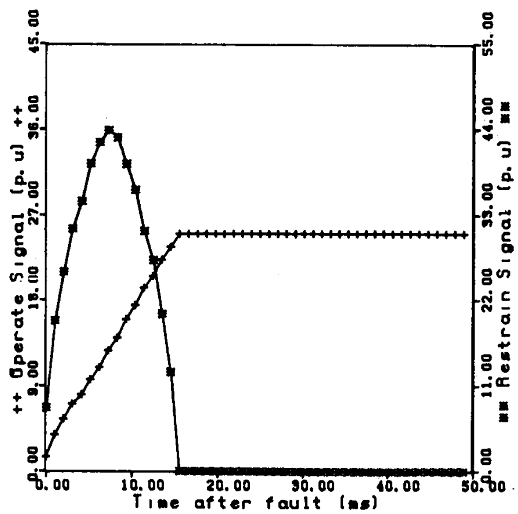

Figure 8.b Rectangular Transform Algorithm Performance for Fault

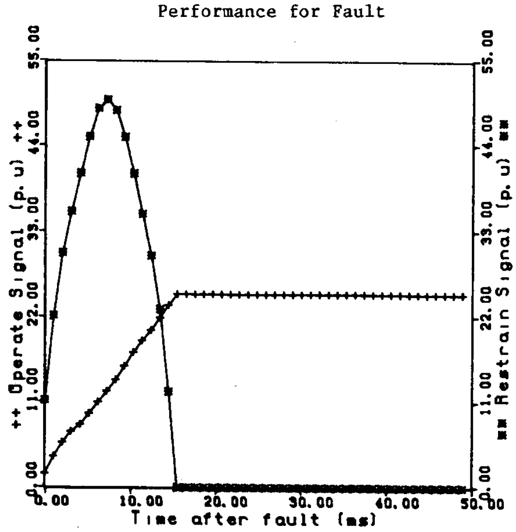

Figure 8.c Walsh Functions Algorithm Performance for Fault

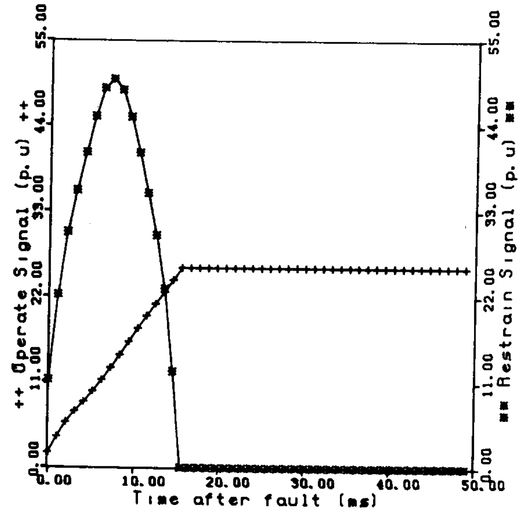

Figure 8.d Haar Functions Algorithm Performance for Fault

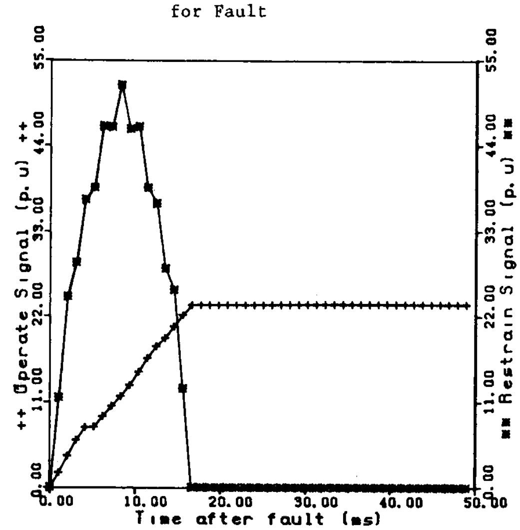

Figure 8.e Finite Impulse Response Algorithm Performance for Fault

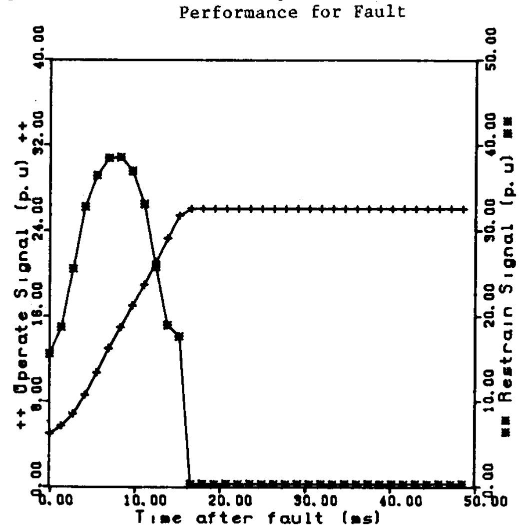

Figure 8.f Curve Fitting Algorithm Performance for Fault

confirmed it issues a trip signal and in case of an inrush tripping is blocked.

A major part of the sampling interval is required for the determination of the fundamental and second harmonic content of the differential currents. The computational requirements for the different algorithms studied are shown in Table 1.

Table 1

Computational Requirements for Transformer Relay Algorithms

Algorithm	Sampling Interval (μs)	Number of Arithmetic operation			Time for Arithmetic Computation (1 Phase) (μs)	Percentage of Sampling Interval
		+/-	x/÷	√		
Fourier	1042	51	14	2	380	36%
Rectangular	1042	106	8	2	410	39%
Walsh	1042	116	14	2	526	50%
Haar	1042	96	16	2	512	49%
Finite	1042	84	4	2	298	29%
Curve	1389	46	19	2	447	32%

The time required for these computations in each sampling interval is also shown. This is based on an INTEL 8086 16 bit microprocessor and INTEL 8087 Numeric Data Processor with a clock frequency of 8 MHz. During the early stages of research in digital relaying the investigators were concerned with replacing the multiplication and division operations by arithmetic shift in order to reduce the time required for the computation. This is not a major concern now since the currently available microprocessors and supporting components can perform high speed arithmetic also.

As may be seen from this table, all algorithms can complete the computations for one phase within the specified sampling interval, but, Curve fitting Fourier and Finite Impulse Response algorithms can do the computations with more than 60% of the time available for data acquisition and relay logic. A major advantage of fast computation is that excess time may be utilized for additional monitoring and control functions. This is very useful if the microprocessor based transformer relay forms part of an integrated microprocessor-based system for relaying and control of substations [23]. The relay designer can decide on the algorithm to be implemented based on the application and the capability of the hardware available.

CONCLUSIONS

Most of the published algorithms for digital differential protection of transformers are compared. All the algorithms are able to distinguish between an inrush and internal fault. In case of an internal fault, the algorithms provide a reliable trip decision within a cycle from the instant of fault occurrence. Based on this study, the Curve fitting, Rectangular Transform, Fourier and Finite Impulse Response algorithms are identified as those which may be easily implemented in a microprocessor based transformer differential relay. These algorithms perform the computations well within the sampling interval. The additional time available in the sampling interval may be utilized for other monitoring and control functions.

REFERENCES

[1] "Computer Relaying Tutorial Text", IEEE Power Engineering Society Publication No. 79EH0148-7-PWR, 1979.

[2] G.B. Gilchrest, G.D. Rockefeller, E.A. Udren, "High-Speed Distance Relaying Using a Digital Computer", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-91, No. 3, May/June 1972, pp. 1235-1258.

[3] M.M. Chen, W.G. Breingan, "Field Experience with Digital System for Transmission Line Protection", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-98, No. 5, Sept./Oct. 1979, pp. 1796-1805.

[4] P. Bornard, J.C. Bastide, "A Prototype of Multiprocessor Based Distance Relay", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-101, No. 2, Feb. 1982, pp. 491-498.

[5] A.L. St. Jaques, G. Santerre, "A Multiprocessor-based Distance Relay: Design Features and Test Results", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-102, No. 12, Dec. 1983, pp. 3842-3849.

[6] J.S. Thorp, A.G. Phadke, "A Microprocessor based Three-Phase Transformer Differential Relay", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-101, No. 2, Feb. 1982, pp. 426-432.

[7] A.J. Degens, J.J.M. Langedijk, "Integral Approach to the Protection of Power Transformers by Means of a Microprocessor", Electrical Power & Energy Systems, Vol. 7, No. 1, Jan. 1985, pp. 37-47.

[8] A.R. van C. Warrington, "Protective Relays; Their Theory and Practice", Vol. 2, Chapman and Hall, London, 1977.

[9] Applied Protective Relaying, Westinghouse Electric Corporation, Newark, N.J., 1976.

[10] J.A. Sykes and I.F. Morrison, "A Proposed Method of Harmonic Restraint Differential Protection of Transformers by Digital Computer", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-91, No. 3, May/June 1972, 1266-1272.

[11] O.P. Malik, P.K. Dash, G.S. Hope, "Digital Protection of a Power Transformer", IEEE Publication 76CH1075-1 PWR, Paper No. A76 191-7, IEEE PES Winter Meeting, New York, January 1976, pp. 1-7.

[12] E.O. Schweitzer, R.R. Larson, A.J. Flechsig Jr., "An Efficient Inrush Current-Detection Algorithm for Digital Computer Relay Protection of Transformers", Paper A77 510-1, IEEE Power Engineering Society Summer Meeting, Mexico City, 1977.

[13] A.J. Degens, "Algorithm for a Digital Transformer Differential Protection Based on a Least-Squares Curve Fitting", Proceedings IEE (London), Vol. 128, Part C, No. 3, May 1981, pp. 155-161.

[14] M.A. Rahman, P.K. Dash, E.R. Downton, "Digital Protection of Power Transformer Based on Weighted Least Square Algorithm", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-101, No. 11, Nov. 1982, pp. 4202-4209.

[15] M.A. Rahman, P.K. Dash, "Fast Algorithm for Digital Protection of Power Transformers", Proceedings IEE(London), Vol. 129, Part C, No. 2, March 1982, pp. 79-85.

[16] D.B. Fakruddin, K. Parthasarathy, L. Jenkins, B.W. Hogg, "Application of Haar Functions for Transmission Line and Transformer Differential Protection", Electrical Power & Energy Systems, Vol. 6, No. 3, July 1984, pp. 169-180.

[17] M.A. Rahman, B. Jeyasurya, A. Gangopadhyay, "Digital Differential Protection of Power Transformers based on Walsh Functions", Transactions of CEA Engineering & Operating Division, Vol. 24, Part 3, Paper 85-SP-149, 1985.

[18] B. Jeyasurya, M.A. Rahman, "Application of Walsh Functions for Microprocessor-Based Transformer Protection", IEEE Trans. on Electromagnetic Compatability, Vol. EMC-27, No. 4, Nov. 1985, pp. 221-225.

[19] W.K. Sonnemann, C.L. Wagner, G.D. Rockefeller, "Magnetizing Inrush Phenomena in Transformer Banks", AIEE Transactions, Vol. 77, October 1958, pp. 884-892.

[20] M.A. Rahman, A. Gangopadhyay, "Digital Simulation of Magnetizing Inrush Currents in Three -Phase Transformers", IEEE Paper No. 86WM 149-9, PES Winter Meeting, New York, Feb. 2-7, 1986.

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[21] T.R. Specht, "Transformer Inrush and Rectifier Transient Currents", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-88, No. 4, April 1969, pp. 269-276.

[22] C.H. Einvall, J.R. Linders, "A Three-Phase Differential Relay for Transformer Protection", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-94, No. 6, Nov./Dec. 1975, pp. 1971-1980.

[23] J.S. Deliyannides, E.A. Udren, "Design Criteria for an Integrated Microprocessor-Based Substation Protection and Control System", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-101, No. 6, June 1982, pp. 1664-1673.

Discussion

M. S. Sachdev, T. S. Sidhu, and **A. Srivastava** (Power System Research Group, University of Saskatchewan, Saskatoon, Sask., Canada S7N 0W0): The paper compares digital relaying algorithms for their application to differential protection of transformers. An attempt has been made to compare the speed of response and ability to discriminate between inrush and internal fault conditions of the selected algorithms. The discussers offer the following comments on the paper.

1) The algorithms have been evaluated for two types of operating conditions. They are a high-energy magnetizing inrush and a severe three-phase internal fault. From the results of these tests, the authors have concluded that the selected algorithms are able to distinguish between magnetizing inrush and internal fault conditions. We are of the opinion that the suitability of an algorithm for implementing in a practical design should be checked by testing the algorithm driving marginal operating conditions. For example, the algorithm should be checked for performance during loads that contain harmonic currents in addition to the fundamental frequency currents.

2) The authors have stated in the section "Simulation Results" that the data window is initialized with samples of zero magnitude before subjecting each algorithm to a magnetizing inrush or fault condition. Because of the initialization of the data windows, the values of restrain and operate signals should be zero at time zero, i.e., at the inception of a fault. However, this is not so in Fig. 8(a), 8(b), 8(c), 8(d), and 8(f) of the paper. Would the authors explain the cause of these discrepancies.

3) It seems to us that the number of some arithmetic operations listed in Table 1 of the paper are not correct. In our opinion, this number of $+/-$ operations for Fourier, Walsh, and Finite Impulse Response algorithms should be 54, 94, and 62, respectively. With these changes in the numbers of $+/-$ operations, the corresponding numbers in columns 6 and 7 of Table 1 would also change.

4) The authors state that the frequency response of two filters for measuring the fundamental frequency and the two filters for measuring the second-harmonic frequency component have been combined to provide single responses shown in Figs. 1-3. We fail to find from the paper how the responses of two filters designed to pass the selected frequency were combined to obtain single responses. In our opinion, combining the outputs of two algorithms to determine the fundamental frequency components provides a band of values. This can be illustrated as follows:

Consider that two algorithms provide orthogonal outputs of $A_s \sin wt$ and $A_c \cos wt$ when the input to the algorithms is $A \sin (wt + \theta)$ as shown in Fig. D.1. Since the filters are nonrecursive, the outputs are orthogonal (they are designed to be) at all frequencies but the peak values of the two outputs are not equal at all frequencies.

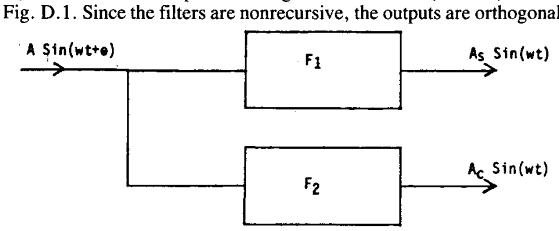

Fig. D.1. Output of orthogonal digital filters.

The outputs of the orthogonal algorithms can be combined as follows to obtain the peak value of the phasor:

$$\begin{aligned} A_e &= [(A_c \sin wt)^2 + (A_s \cos wt)^2]^{0.5} \\ &= [A_c^2 \sin^2 wt + A_s^2 \cos^2 wt]^{0.5} \\ &= \frac{1}{2} [(A_s^2 + A_c^2) + (A_s^2 - A_c^2) \cos 2wt]^{0.5}. \end{aligned}$$

The outputs are time-invariant for all frequencies for which A_s is equal to A_c. At all other frequencies, the outputs are time-variant. If A_s is greater than A_c, A_e would have a maximum value of A_s at $wt = 0, 2\pi, \cdots$, and a minimum value of A_c at $wt = \pi, 3\pi, \cdots$. Similarly, if A_c is greater than A_s, A_e would have a maximum value of A_c and a minimum value of A_s. The estimated peak value of the phasor would therefore be in the range A_s and A_c as shown in Fig. D.2. It is interesting to note that the average value

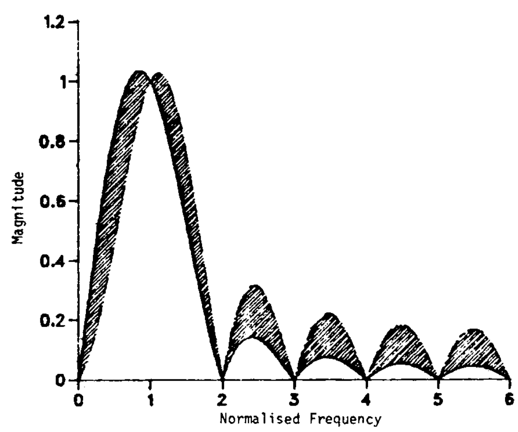

Fig. D.2. Range of estimated values of the phasor.

of A_e^2 is $0.5(A_s^2 + A_c^2)$ but the average value of A_e is not $[\frac{1}{2}(A_s^2 + A_c^2)]^{1/2}$. A similar phenomenon is observed, when the responses for the second-harmonic filters are combined.

5) We would also like to point out that the Fourier Algorithm should have been implemented using 12 samples per cycle. This would have required fewer computations in comparison to the author's approach of using 16 samples per cycle. Twelve sample Fourier algorithm requires only 42 additions/subtractions, 8 multiplications, and 2 square root operations.

6) We would also like to point out that on-line application of the least square error approach was developed at University of Saskatchewan in the 1970's [1]. The algorithm was used to design an algorithm for distance protection of transmission lines. Later, the least square error approach was applied to differential and earth fault protection of transformers [2].

References

[1] M. S. Sachdev and M. A. Baribeau, "A new algorithm for digital impedence relays," *IEEE Trans. Power App. Syst.*, vol. PAS-98, no. 6, pp. 2232–2240, Nov./Dec. 1979.

[2] M. S. Sachdev and D. V. Shah, "Transformer differential and restricted earth fault protection using a digital processor," in *Trans. CEA,* vol. 20, no. 4, paper 81-SP-155, Mar. 1981.

Manuscript received March 3, 1987.

M. A. Rahman and **B. Jeyasurya**: The authors thank the discussers for their comments on the paper. In fact, Prof. M. S. Sachdev and his associates have extended the paper by the discussion contribution.

1) The authors agree with the discussers that performance of the algorithms should also be checked during marginal operating conditions. Since the objective of the paper was to compare the algorithms with respect to their capability to distinguish between an inrush and internal fault, saturation of current transformers, loads containing additional harmonic currents, etc., have not been simulated. A practical design implementing any suitable algorithm should incorporate additional logic to ensure that the relay provides a secure restraining signal during operating conditions which call for undesirable trip signal ([7] of the paper).

2) The filter coefficients of the algorithms corresponding to the performance shown in Figs. 8(a), 8(b), 8(c), 8(d), and 8(f) of the paper have finite values at the beginning of the data window. At $t = 0^+$, the fault current also has a finite value. This is multiplied by the finite filter coefficient resulting in finite values of restraint and operating signals.

3) The arithmetic operations listed in Table 1 of the paper are arrived at after simulating a transformer digital relay, implementing the various algorithms on a Personal Computer. The computational requirements are based on an INTEL 8086 Microprocessor and an INTEL 8087 Numeric Data Processor which is required to do multiply, divide, and square root operations. The sampling rate is 16 samples per cycle. The programs written in Assembly language are available from the authors from which one may verify the total number of required arithmetic operations. The number of arithmetic operations is correct. However, it is to be noted that the number of operations can be reduced further. But it requires additional MOVE, STORE, and LOAD instructions which are more time-consuming than simple ADD/SUB instructions.

4) The frequency response plot for an algorithm is obtained by transforming the algorithm into the complex Z-domain and substituting frequency for the complex variable $Z(Z = \omega T$, where T is the sampling interval). The outputs are time-invariant for a specified sampling interval as per [1] of the paper and [A]. The responses shown in Figs. 1–3 of the paper are obtained by evaluating the square root of the sum of the squares of the individual responses. The discussers provided a valid argument on two orthogonal algorithms. In the paper it is assumed that A_s is equal to A_c. Thus it is implied that both the filter outputs are time-invariant. For the sake of completeness, the typographical errors of the discussion are corrected as follows

$$\begin{aligned} A_e &= [(A_s \sin \omega t)^2 + (A_c \cos \omega t)^2]^{0.5} \\ &= [A_s^2 \sin^2 \omega t + A_c^2 \cos^2 \omega t)]^{0.5} \end{aligned}$$

The output of filter F_2 is $A_c \cos \omega t$.

5) The twelve-sample Fourier algorithm will reduce the total computational requirements as pointed out by the discussers. Some of the algorithms studied specifically require that the number of samples per cycle be an integral power of 2. Hence a rate of 16 samples per cycle was also considered for the Fourier algorithm.

6) The authors took note of the discussers' research on the application of the least square error approach for digital protection as noted in [14] of the paper.

Reference

[A] M. Habib and M. A. Marin, "A comparative analysis of digital relaying algorithms for the differential protection of three phase transformers," in *IEEE 1987 PICA Conf. Rec.* (Montreal, Que., Canada, 1987), pp. 460–466.

Manuscript received June 30, 1987.