SPUR GEARS
GEAR NOMENCLATURE

ADDENDUM (a) is the height by which a tooth projects beyond the pitch circle or pitch line.

BASE DIAMETER (D_b) is the diameter of the base cylinder from which the involute portion of a tooth profile is generated.

BACKLASH (B) is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles. As actually indicated by measuring devices, backlash may be determined variously in the transverse, normal, or axial-planes, and either in the direction of the pitch circles or on the line of action. Such measurements should be corrected to corresponding values on transverse pitch circles for general comparisons.

BORE LENGTH is the total length through a gear, sprocket, or coupling bore.

CIRCULAR PITCH (p) is the distance along the pitch circle or pitch line between corresponding profiles of adjacent teeth.

CIRCULAR THICKNESS (t) is the length of arc between the two sides of a gear tooth on the pitch circle, unless otherwise specified.

CLEARANCE-OPERATING (c) is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear.

CONTACT RATIO (m_c) in general, the number of angular pitches through which a tooth surface rotates from the beginning to the end of contact.

DEDENDUM (b) is the depth of a tooth space below the pitch line. It is normally greater than the addendum of the mating gear to provide clearance.

DIAMETRAL PITCH (P) is the ratio of the number of teeth to the pitch diameter.

FACE WIDTH (F) is the length of the teeth in an axial plane.

FILLET RADIUS (r_f) is the radius of the fillet curve at the base of the gear tooth.

FULL DEPTH TEETH are those in which the working depth equals 2.000 divided by the normal diametral pitch.

GEAR is a machine part with gear teeth. When two gears run together, the one with the larger number of teeth is called the gear.

HUB DIAMETER is outside diameter of a gear, sprocket or coupling hub.

HUB PROJECTION is the distance the hub extends beyond the gear face.

INVOLUTE TEETH of spur gears, helical gears and worms are those in which the active portion of the profile in the transverse plane is the involute of a circle.

LONG- AND SHORT-ADDENDUM TEETH are those of engaging gears (on a standard designed center distance) one of which has a long addendum and the other has a short addendum.

KEYWAY is the machined groove running the length of the bore. A similar groove is machined in the shaft and a key fits into this opening.

NORMAL DIAMETRAL PITCH (P_n) is the value of the diametral pitch as calculated in the normal plane of a helical gear or worm.

NORMAL PLANE is the plane normal to the tooth surface at a pitch point and perpendicular to the pitch plane. For a helical gear this plane can be normal to one tooth at a point laying in the plane surface. At such point, the normal plane contains the line normal to the tooth surface and this is normal to the pitch circle.

NORMAL PRESSURE ANGLE ($ø_n$) in a normal plane of helical tooth.

OUTSIDE DIAMETER (D_o) is the diameter of the addendum (outside) circle.

SPUR GEARS
GEAR NOMENCLATURE (Continued)

PITCH CIRCLE is the circle derived from a number of teeth and a specified diametral or circular pitch. Circle on which spacing or tooth profiles is established and from which the tooth proportions are constructed.

PITCH CYLINDER is the cylinder of diameter equal to the pitch circle.

PINION is a machine part with gear teeth. When two gears run together, the one with the smaller number of teeth is called the pinion.

PITCH DIAMETER (D) is the diameter of the pitch circle. In parallel shaft gears, the pitch diameters can be determined directly from the center distance and the number of teeth.

PRESSURE ANGLE (ø) is the angle at a pitch point between the line of pressure which is normal to the tooth surface, and the plane tangent to the pitch surface. In involute teeth, pressure angle is often described also as the angle between the line of action and the line tangent to the pitch circle. Standard pressure angles are established in connection with standard gear-tooth proportions.

ROOT DIAMETER (D_r) is the diameter at the base of the tooth space.

PRESSURE ANGLE—OPERATING ($ø_r$) is determined by the center distance at which the gears operate. It is the pressure angle at the operating pitch diameter.

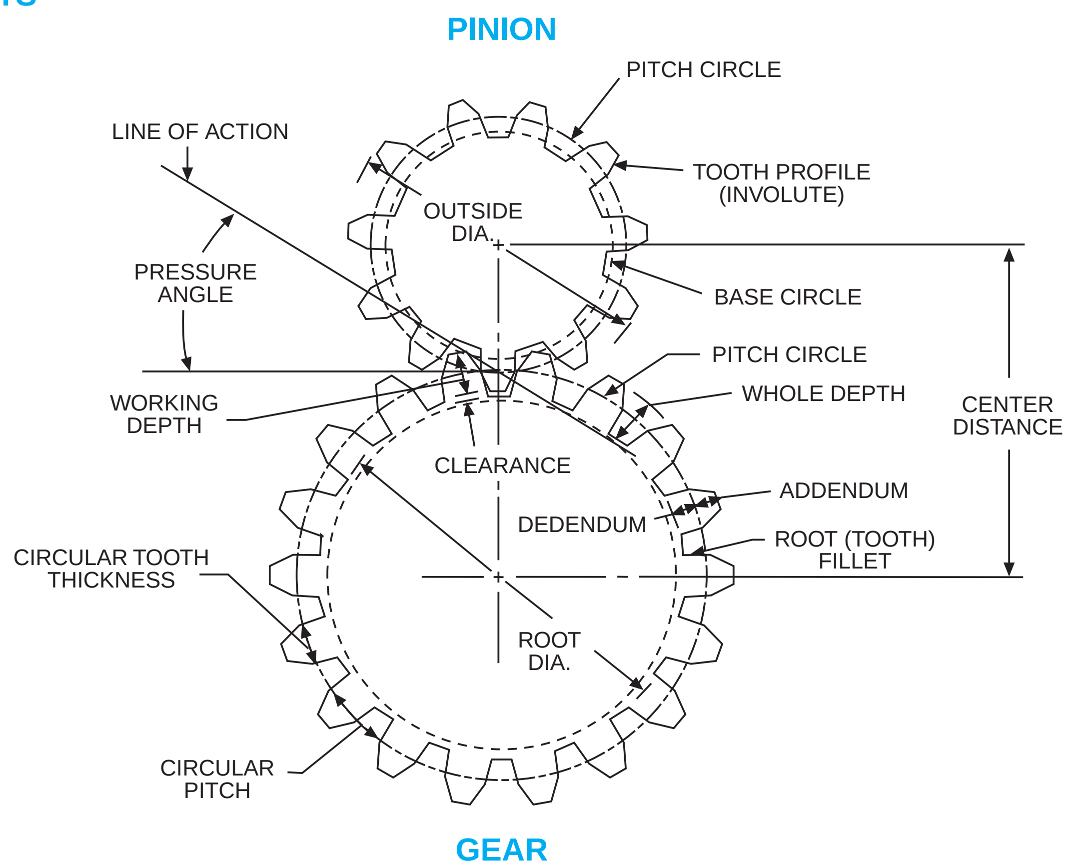
TIP RELIEF is an arbitrary modification of a tooth profile whereby a small amount of material is removed near the tip of the gear tooth.

UNDERCUT is a condition in generated gear teeth when any part of the fillet curve lies inside a line drawn tangent to the working profile at its point of juncture with the fillet.

WHOLE DEPTH (h_t) is the total depth of a tooth space, equal to addendum plus dedendum, equal to the working depth plus variance.

WORKING DEPTH (h_k) is the depth of engagement of two gears; that is, the sum of their addendums.

TOOTH PARTS

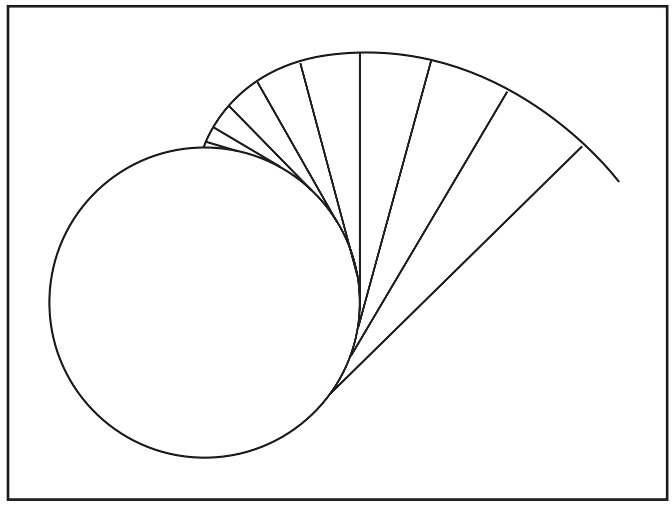


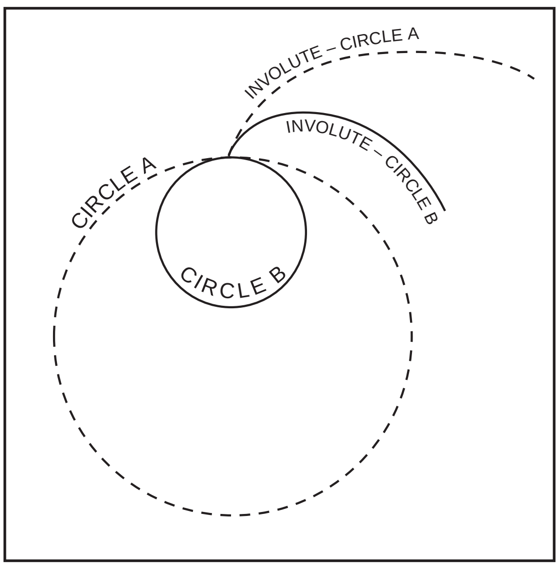
SPUR GEARS
INVOLUTE FORM

Gear teeth could be manufactured with a wide variety of shapes and profiles. The involute profile is the most commonly used system for gearing today, and all Boston spur and helical gears are of involute form.

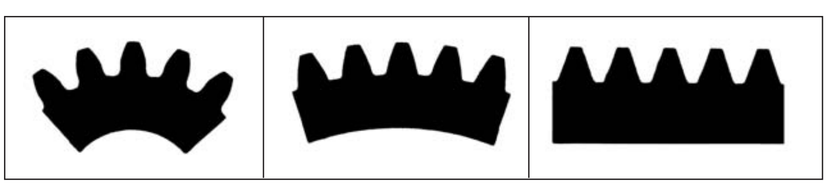
An involute is a curve that is traced by a point on a taut cord unwinding from a circle, which is called a BASE CIRCLE. The involute is a form of spiral, the curvature of which becomes straighter as it is drawn from a base circle and eventually would become a straight line if drawn far enough.

An involute drawn from a larger base circle will be less curved (straighter) than one drawn from a smaller base circle. Similarly, the involute tooth profile of smaller gears is considerably curved, on larger gears is less curved (straighter), and is straight on a rack, which is essentially an infinitely large gear.





Involute gear tooth forms and standard tooth proportions are specified in terms of a basic rack which has straight-sided teeth, for involute systems.



20 TEETH　48 TEETH　RACK

SPUR GEARS

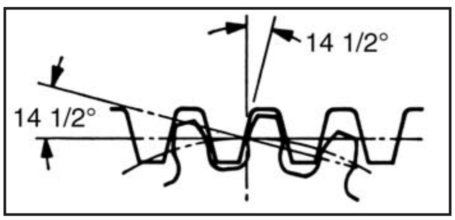
DIAMETRAL PITCH SYSTEM

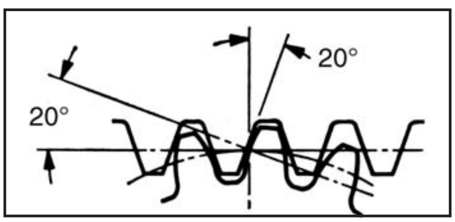
All stock gears are made in accordance with the diametral pitch system. The diametral pitch of a gear is the number of teeth in the gear for each inch of pitch diameter. Therefore, the diametral pitch determines the size of the gear tooth.

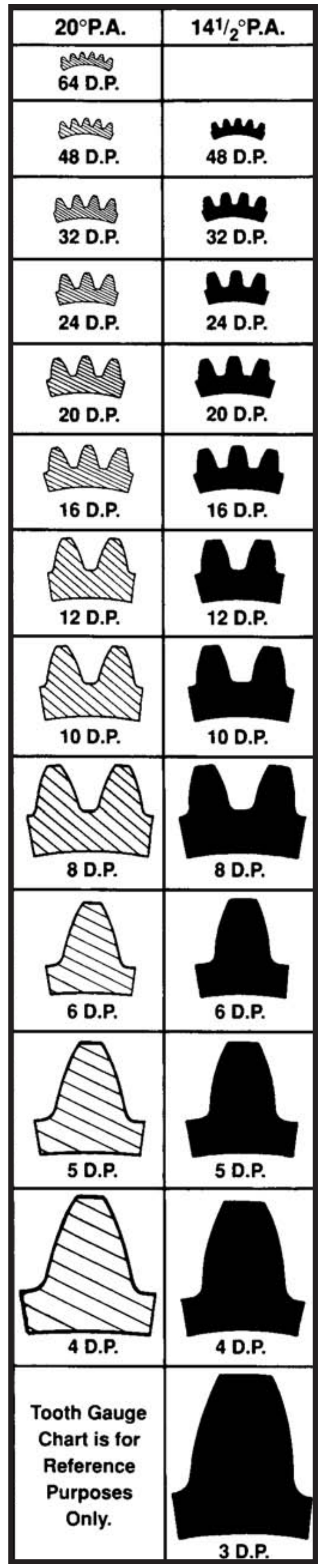
PRESSURE ANGLE

Pressure angle is the angle at a pitch point between the line of pressure which is normal to the tooth surface, and the plane tangent to the pitch surface. The pressure angle, as defined in this catalog, refers to the angle when the gears are mounted on their standard center distances.

Boston Gear manufactures both 14-1/2° and 20° PA, involute, full depth system gears. While 20°PA is generally recognized as having higher load carrying capacity, 14-1/2°PA gears have extensive use. The lower pressure angle results in less change in backlash due to center distance variation and concentricity errors. It also provides a higher contact ratio and consequent smoother, quieter operation provided that undercut of teeth is not present.







TOOTH DIMENSIONS

For convenience, Tooth Proportions of various standard diametral pitches of Spur Gears are given below.

Diametral Pitch	Circular Pitch (Inches)	Thickness of Tooth on Pitch Line (Inches)	Depth to be Cut in Gear (Inches) (Hobbed Gears)	Addendum (Inches)
3	1.0472	.5236	.7190	.3333
4	.7854	.3927	.5393	.2500
5	.6283	.3142	.4314	.2000
6	.5236	.2618	.3565	.1667
8	.3927	.1963	.2696	.1250
10	.3142	.1571	.2157	.1000
12	.2618	.1309	.1798	.0833
16	.1963	.0982	.1348	.0625
20	.1571	.0785	.1120	.0500
24	.1309	.0654	.0937	.0417
32	.0982	.0491	.0708	.0312
48	.0654	.0327	.0478	.0208
64	.0491	.0245	.0364	.0156

SPUR GEARS

BACKLASH

Stock spur gears are cut to operate at standard center distances. The standard center distance being defined by:

Standard Center Distance = $\frac{\text{Pinion PD + Gear PD}}{2}$

When mounted at this center distance, stock spur gears will have the following average backlash:

Diametral Pitch	Backlash (Inches)	Diametral Pitch	Backlash (Inches)
3	.013	8-9	.005
4	.010	10-13	.004
5	.008	14-32	.003
6	.007	33-64	.0025
7	.006		

An increase or decrease in center distance will cause an increase or decrease in backlash.

Since, in practice, some deviation from the theoretical standard center distance is inevitable and will alter the backlash, such deviation should be as small as possible. For most applications, it would be acceptable to limit the deviation to an increase over the nominal center distance of one half the average backlash. Varying the center distance may afford a practical means of varying the backlash to a limited extent.

The approximate relationship between center distance and backlash change of 14-1/2° and 20° pressure angle gears is shown below:

For 14-1/2°–Change in Center Distance = 1.933 x Change in Backlash
For 20° –Change in Center Distance = 1.374 x Change in Backlash

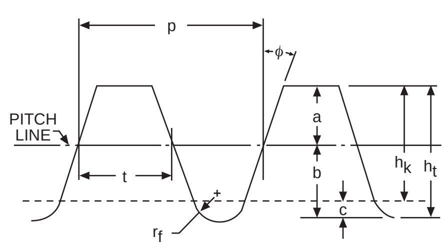
From this, it is apparent that a given change in center distance, 14-1/2° gears will have a smaller change in backlash than 20° gears. This fact should be considered in cases where backlash is critical.

UNDERCUT

When the number of teeth in a gear is small, the tip of the mating gear tooth may interfere with the lower portion of the tooth profile. To prevent this, the generating process removes material at this point. This results in loss of a portion of the involute adjacent to the tooth base, reducing tooth contact and tooth strength.

On 14-1/2°PA gears undercutting occurs where a number of teeth is less than 32 and for 20°PA less than 18. Since this condition becomes more severe as tooth numbers decrease, it is recommended that the minimum number of teeth be 16 for 14-1/2°PA and 13 for 20°PA.

In a similar manner INTERNAL Spur Gear teeth may interfere when the pinion gear is too near the size of its mating internal gear. The following may be used as a guide to assure proper operation of the gear set. For 14-1/2°PA, the difference in tooth numbers between the gear and pinion should not be less than 15. For 20°PA the difference in tooth numbers should not be less than 12.



a = ADDENDUM
b = DEDENDUM
c = CLEARANCE
h_k = WORKING DEPTH
h_t = WHOLE DEPTH
p = CIRCULAR PITCH
r_f = FILLET RADIUS
t = CIRCULAR TOOTH THICKNESS
φ = PRESSURE ANGLE

SPUR GEAR FORMULAS

FOR FULL DEPTH INVOLUTE TEETH

To Obtain	Having	Formula
Diametral Pitch (P)	Circular Pitch (p)	$P = \frac{3.1416}{p}$
	Number of Teeth (N) & Pitch Diameter (D)	$P = \frac{N}{D}$
	Number of Teeth (N) & Outside Diameter (D_o)	$P = \frac{N + 2}{D_o}$ (Approx.)
Circular Pitch (p)	Diametral Pitch (P)	$p = \frac{3.1416}{P}$
Pitch Diameter (D)	Number of Teeth (N) & Diametral Pitch (P)	$D = \frac{N}{P}$
	Outside Diameter (D_o) & Diametral Pitch (P)	$D = D_o - \frac{2}{P}$
Base Diameter (D_b)	Pitch Diameter (D) and Pressure Angle (ø)	Db = Dcosø
Number of Teeth (N)	Diametral Pitch (P) & Pitch Diameter (D)	N = P x D
Tooth Thickness (t) @Pitch Diameter (D)	Diametral Pitch (P)	$t = \frac{1.5708}{P}$
Addendum (a)	Diametral Pitch (P)	$a = \frac{1}{P}$
Outside Diameter (D_o)	Pitch Diameter (D) & Addendum (a)	$D_o = D + 2a$
Whole Depth (h_t) (20P & Finer)	Diametral Pitch (P)	$h_t = \frac{2.2}{P} + .002$
Whole Depth (h_t) (Courser than 20P)	Diametral Pitch (P)	$h_t = \frac{2.157}{P}$
Working Depth (h_k)	Addendum (a)	$h_k = 2(a)$
Clearance (c)	Whole Depth (h_t) Addendum (a)	$c = h_t - 2a$
Dedendum (b)	Whole Depth (h_t) & Addendum (a)	$b = h_t - a$
Contact Ratio (M_c)	Outside Radii, Base Radii, Center Distance and Pressure Angle+C.P.	
$M_c = \frac{\sqrt{R_o^2 - R_b^2} + \sqrt{r_o^2 - r_b^2} - C\sin ø^*}{p \cos ø}$		
Root Diameter (D_r)	Pitch Diameter (D) and Dedendum (b)	$D_r = D - 2b$
Center Distance (C)	Pitch Diameter (D) or No. of Teeth and Pitch	$C = \frac{D_1 + D_2}{2}$ or $\frac{N_1 + N_2}{2P}$

*R_o = Outside Radius, Gear
r_o = Outside Radius, Pinion
R_b = Base Circle Radius, Gear
r_b = Base Circle Radius, Pinion

SPUR GEARS

LEWIS FORMULA (Barth Revision)

Gear failure can occur due to tooth breakage (tooth stress) or surface failure (surface durability) as a result of fatigue and wear. Strength is determined in terms of tooth-beam stresses for static and dynamic conditions, following well established formula and procedures. Satisfactory results may be obtained by the use of Barth's Revision to the Lewis Formula, which considers beam strength but not wear. The formula is satisfactory for commercial gears at Pitch Circle velocities of up to 1500 FPM. It is this formula that is the basis for all Boston Spur Gear ratings.

METALLIC SPUR GEARS

$$W = \frac{SFY}{P}\left(\frac{600}{600 + V}\right)$$

W = Tooth Load, Lbs. (along the Pitch Line)
S = Safe Material Stress (static) Lbs. per Sq. In. (Table II)
F = Face Width, In.
Y = Tooth Form Factor (Table I)
P = Diametral Pitch
D = Pitch Diameter
V = Pitch Line Velocity, Ft. per Min. = .262 x D x RPM

For NON-METALLIC GEARS, the modified Lewis Formula shown below may be used with (S) values of 6000 PSI for Phenolic Laminated material.

$$W = \frac{SFY}{P}\left(\frac{150}{200 + V} + .25\right)$$

TABLE II–VALUES OF SAFE STATIC STRESS (s)

Material		(s) Lb. per Sq. In.
Plastic		5000
Bronze		10000
Cast Iron		12000
Steel	.20 Carbon (Untreated)	20000
Steel	.20 Carbon (Case-hardened)	25000
Steel	.40 Carbon (Untreated)	25000
Steel	.40 Carbon (Heat-treated)	30000
Steel	.40 C. Alloy (Heat-treated)	40000

Max. allowable torque (T) that should be imposed on a gear will be the safe tooth load (W) multiplied by $\frac{D}{2}$ or $T = \frac{W \times D}{2}$

The safe horsepower capacity of the gear (at a given RPM) can be calculated from $HP = \frac{T \times RPM}{63{,}025}$ or directly from (W) and (V);

$$HP = \frac{WV}{33{,}000}$$

For a known HP, $T = \frac{63025 \times HP}{RPM}$

TABLE I TOOTH FORM FACTOR (Y)

Number of Teeth	14-1/2° Full Depth Involute	20° Full Depth Involute
10	0.176	0.201
11	0.192	0.226
12	0.210	0.245
13	0.223	0.264
14	0.236	0.276
15	0.245	0.289
16	0.255	0.295
17	0.264	0.302
18	0.270	0.308
19	0.277	0.314
20	0.283	0.320
22	0.292	0.330
24	0.302	0.337
26	0.308	0.344
28	0.314	0.352
30	0.318	0.358
32	0.322	0.364
34	0.325	0.370
36	0.329	0.377
38	0.332	0.383
40	0.336	0.389
45	0.340	0.399
50	0.346	0.408
55	0.352	0.415
60	0.355	0.421
65	0.358	0.425
70	0.360	0.429
75	0.361	0.433
80	0.363	0.436
90	0.366	0.442
100	0.368	0.446
150	0.375	0.458
200	0.378	0.463
300	0.382	0.471
Rack	0.390	0.484

HELICAL GEARS

GEAR NOMENCLATURE

The information contained in the Spur Gear section is also pertinent to Helical Gears with the addition of the following:

HELIX ANGLE (ψ) is the angle between any helix and an element of its cylinder. In helical gears, it is at the pitch diameter unless otherwise specified.

LEAD (L) is the axial advance of a helix for one complete turn, as in the threads of cylindrical worms and teeth of helical gears.

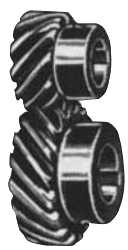
NORMAL DIAMETRAL PITCH (P_n) is the Diametral Pitch as calculated in the normal plane.

HAND – Helical Gears of the same hand operate at right angles, see Fig. 1
Helical Gears of opposite hands run on parallel shafts. Fig. 2

TWO RIGHT-HAND HELICAL GEARS

TWO LEFT-HAND HELICAL GEARS

Figure 1



LEFT-HAND AND RIGHT-HAND HELICAL GEARS

Figure 2

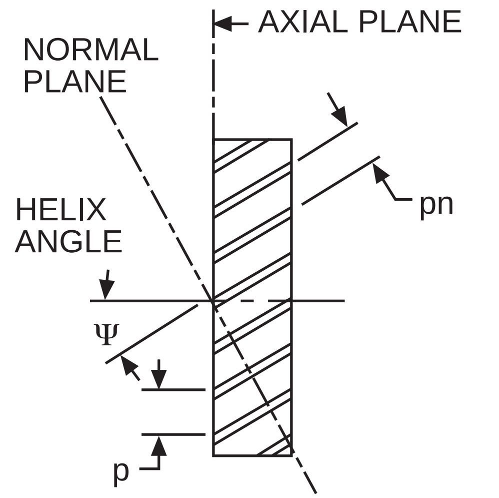
LEFT HAND HELICAL GEAR

The teeth of a LEFT HAND Helical Gear lean to the left when the gear is placed flat on a horizontal surface.

RIGHT HAND HELICAL GEAR

The teeth of a RIGHT HAND Helical Gear lean to the right when the gear is placed flat on a horizontal surface.

HELIX ANGLE—



All Boston Helicals are cut to the Diametral Pitch system, resulting in a Normal Pitch which is lower in number than the Diametral Pitch.

INVOLUTE—The Helical tooth form is involute in the plane of rotation and can be developed in a manner similar to that of the Spur Gear. However, unlike the Spur Gear, which may be viewed as two-dimensional, the Helical Gear must be viewed as three-dimensional to show change in axial features.

Helical gears offer additional benefits relative to Spur Gears, those being:

- Improved tooth strength due to the elongated helical wrap-around.
- Increased contact ratio due to the axial tooth overlap.
- Helical Gears thus tend to have greater load carrying capacity than Spur Gears of similar size.
- Due to the above, smoother operating characteristics are apparent.

ENGINEERING INFORMATION

HELICAL GEARS

HELICAL GEAR FORMULAS

To Obtain	Having	Formula
Transverse Diametral Pitch (P)	Number of Teeth (N) & Pitch Diameter (D)	$P = \frac{N}{D}$
	Normal Diametral Pitch (P_n) Helix Angle (ψ)	$P = P_N \cos\psi$
Pitch Diameter (D)	Number of Teeth (N) & Transverse Diametral Pitch (P)	$D = \frac{N}{P}$
Normal Diametral Pitch (P_N)	Transverse Diametral Pitch (P) & Helix Angle (ψ)	$P_N = \frac{P}{\cos\psi}$
Normal Circular Tooth Thickness (τ)	Normal Diametral Pitch (P_N)	$\tau = \frac{1.5708}{P_N}$
Transverse Circular Pitch (p_t)	Diametral Pitch (P) (Transverse)	$p_t = \frac{\pi}{P}$
Normal Circular Pitch (p_n)	Transverse Circular Pitch (p)	$p_n = p_t \cos\psi$
Lead (L)	Pitch Diameter and Pitch Helix Angle	$L = \frac{\pi D}{\tan\psi}$

TRANSVERSE VS. NORMAL DIAMETRAL PITCH FOR BOSTON 45° HELICAL GEARS

P Transverse Diametral Pitch	P_N Normal Diametral Pitch
24	33.94
20	28.28
16	22.63
12	16.97
10	14.14
8	11.31
6	8.48

HELICAL GEAR LEWIS FORMULA

The beam strength of Helical Gears operating on *parallel shafts* can be calculated with the Lewis Formula revised to compensate for the difference between Spur and Helical Gears, with modified Tooth Form Factors Y.

$$W = \frac{SFY}{P_N}\left(\frac{600}{600 + V}\right)$$

W = Tooth Load, Lbs. (along the Pitch Line)
S = Safe Material Stress (static) Lbs. per Sq. In. (Table III)
F = Face Width, Inches
Y = Tooth Form Factor (Table IV)
P_N = Normal Diametral Pitch
(Refer to Conversion Chart)
D = Pitch Diameter
V = Pitch Line Velocity, Ft. Per Min. = .262 x D x RPM

TABLE III–VALUES OF SAFE STATIC STRESS (S)

Material		(s) Lb. per Sq. In.
Bronze		10000
Cast Iron		12000
Steel	.20 Carbon (Untreated)	20000
	.20 Carbon (Case-hardened)	25000
	.40 Carbon (Untreated)	25000
	.40 Carbon (Heat-treated)	30000
	.40 C. Alloy (Heat-treated)	40000

TABLE IV—VALUES OF TOOTH FORM FACTOR (Y)

FOR 14-1/2°PA—45° HELIX ANGLE GEAR			
No. of Teeth	Factor Y	No. of Teeth	Factor Y
8	.295	25	.361
9	.305	30	.364
10	.314	32	.365
12	.327	36	.367
15	.339	40	.370
16	.342	48	.372
18	.345	50	.373
20	.352	60	.374
24	.358	72	.377

HORSEPOWER AND TORQUE

Max. allowable torque (T) that should be imposed on a gear will be the safe tooth load (W) multiplied by $\frac{D}{2}$ or $T = \frac{W \times D}{2}$

The safe horsepower capacity of the gear (at a given RPM) can be calculated from $HP = \frac{T \times RPM}{63,025}$ or directly from (W) and (V);

$$HP = \frac{WV}{33,000}$$

For a known HP, $T = \frac{63025 \times HP}{RPM}$

HELICAL GEARS

When Helical gears are operated on other than Parallel shafts, the tooth load is concentrated at a point, with the result that very small loads produce very high pressures. The sliding velocity is usually quite high and, combined with the concentrated pressure, may cause galling or excessive wear, especially if the teeth are not well lubricated. For these reasons, the tooth load which may be applied to such drives is very limited and of uncertain value, and is perhaps best determined by trial under actual operating conditions. If one of the gears is made of bronze, the contact area and thereby the load carrying capacity, may be increased, by allowing the gears to "run-in" in their operating position, under loads which gradually increase to the maximum expected.

THRUST LOADS

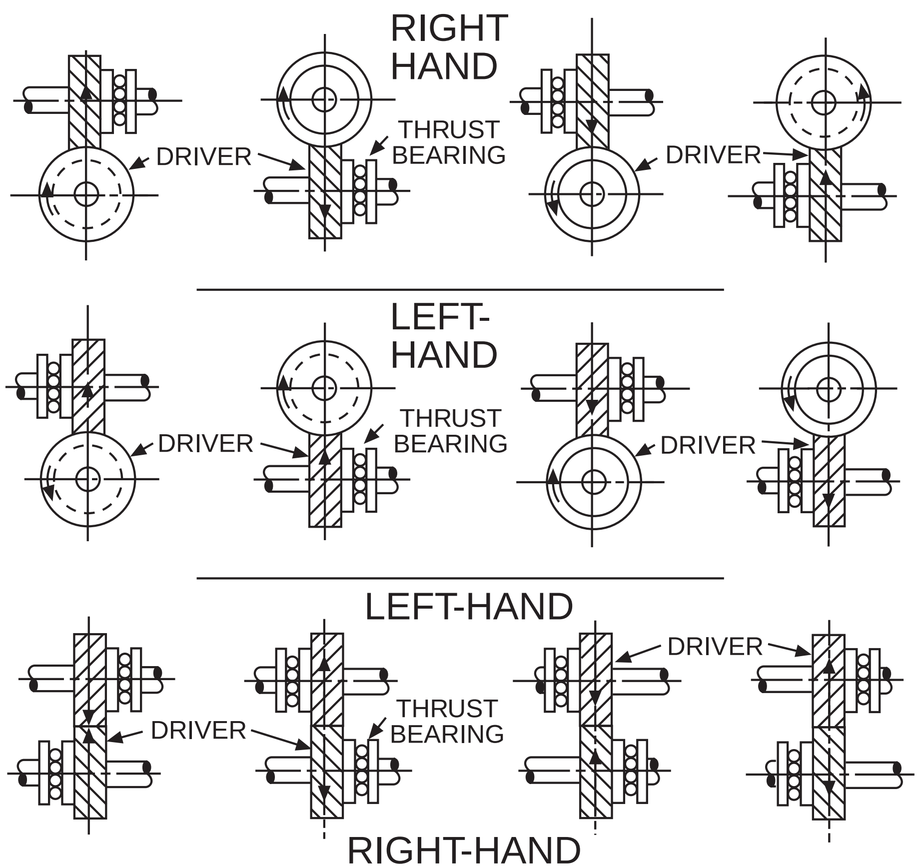
As a result of the design of the Helical Gear tooth, an axial or thrust load is developed. Bearings must be adequate to absorb this load. The thrust load direction is indicated below. The magnitude of the thrust load is based on calculated Horsepower.

$$\text{Axial Thrust Load} = \frac{126{,}050 \times \text{HP}}{\text{RPM} \times \text{Pitch Diameter}}$$

Boston Helicals are all 45° Helix Angle, producing a tangential force equal in magnitude to the axial thrust load. A separating force is also imposed on the gear set based on calculated Horsepower.

Separating Load = Axial Thrust Load x .386

Above formulae based on Boston 45° Helix Angle and 14-1/2° Normal Pressure Angle.



See page 118 for hardened and ground Thrust Washers.